Honors Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@uchicago.edu
Course Homepage: people.cs.uchicago.edu/~razborov/teaching/autumn23.html

Autumn Quarter, 2023

`Prove all of your answers with reasonable degree of mathematical rigor (feel free to ask us when in doubt). If you work with others put their names clearly at the top of the assignment, everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged. But if you accidentally stumble across it, then it will be accepted as long as you cite the source and explain the solution in your own words.`

Homework 5, due November 9

1. Prove that for any integers $m, n \geq 1$, $\frac{(mn)!}{(n!)^m \cdot m!}$ is also an integer.

2. How many ways of distributing the standard 32 chess pieces into four distinguishable pouches are there? Pieces of the same kind and color (like two black rooks) are considered indistinguishable; the pouches are allowed to be empty.

   `Note: in this and the next problem we want a plain integer as the final answer. But once you write down a closed form expression, you may just evaluate it using a calculator without further explanations.`

3. How many solutions are there to the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 20,$$

   where x_i are *odd* positive integers? Order matters.

4. Prove the recursion

$$p_n(m) = p_n(m-n) + p_{n-1}(m) \ (m \geq n),$$

where $p_n(m)$ is the number of partitions of m using at most n numbers.

5. Six points are chosen inside a unit disk. Prove that there is a pair of them at the distance ≤ 1 of each other.