Quantum Computing

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Homework 1, due May 7

1. Prove that the TOFFOLI gate can not be realized as a reversible circuit with gates acting on at most two bits.
   
   Hint. Reversible gates on two bits have one very special property; identify it.

2. For which values $N \leq 11$ does there exist an $N \times N$ unitary matrix in which all entries are $\pm \frac{1}{\sqrt{N}}$?

3. We have seen in class how to implement the oracle $V_f : V_f |x\rangle = (-1)^{f(x)} |x\rangle$ using the oracle $U_f : U_f |x, y\rangle = |x, y \oplus f(x)\rangle$.

   (a) Prove that the opposite is impossible.
   
   (b) Implement $U_f$ using the controlled version of $V_f$, i.e., the operator given by
   
   $\Lambda(V_f) : |x, y\rangle \mapsto \begin{cases} |x, y\rangle & \text{if } y = 0 \\ (-1)^{f(x)} |x, y\rangle & \text{if } y = 1 \end{cases}$

4. Prove that an $N$-dimensional operator $A$ is normal if and only if $A^\dagger$ can be represented as a linear combination of $I_N, A, A^2, \ldots, A^{N-1}$.

   Hint. The solution I have in mind uses Lagrange’s interpolation. But you should feel free to shop for a solution in textbooks on linear algebra as well.

5. Prove that $QFT_{mn} \approx QFT_m \otimes QFT_n$ if and only if $m$ and $n$ are relatively prime. Here $\approx$ stands for the unitary equivalence: $A \approx B$ if and only if there exists a unitary operator $P$ such that $B = PAP^\dagger$. 

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