Quantum Computing

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Course Homepage: www.cs.uchicago.edu/~ razborov/teaching/winter20.html

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You may work together on solving homework problems, but please put all the names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged but if you do it anyway, you must completely understand the proof, explain it in your own words and include the URL.

Handwritten submissions must be turned in on the due date below before the class. PDF file **prepared from a TeX source** is much preferred format and such submissions are entitled to automatic extension until Friday morning.

Homework 1, due February 6

1. Prove that there are only $\exp(O(n^2))$ permutations $f : \{0,1\}^n \longrightarrow \{0,1\}^n$ that can be realized by a reversible circuit with reversible gates on ≤ 2 bits.

2. Let U and V be linear operators in finite-dimensional Hilbert spaces. Prove that $U \otimes V$ is unitary if and only if there exists a positive real number α such that both αU and $\alpha^{-1}V$ are unitary.

3. For which values $N \leq 11$ does there exist an $N \times N$ unitary matrix in which all entries are $\pm\frac{1}{\sqrt{N}}$?

4. Recall that an alternative way of giving access to a Boolean function $f : \{0,1\}^n \longrightarrow \{0,1\}$ is via the phase-flipping black box U_f^* given by $U_f^* : |x\rangle \mapsto (-1)^{f(x)}|x\rangle$.

   Let $n = 2$. Prove that there is no quantum circuit in this model computing the predicate $f(0,0) \oplus f(0,1) \oplus f(1,0)$.

5. Prove that any polynomial size quantum circuit over the standard basis $\{H, \text{CNOT}, K, \text{TOFFOLI}\}$ that uses only $O(\log n)$ Hadamard gates can be evaluated by a polynomial time algorithm.