Quantum Computing

Instructor: Alexander Razborov, University of Chicago.
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~ razborov/teaching/winter20.html

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`You may work together on solving homework problems, but please put all the names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged but if you do it anyway (or just unintentionally came across it), you must completely understand the proof, explain it in your own words and include the URL.`

Homework 2, due February 26

1. Prove that any **zero-error** quantum algorithm for searching a database of N entries must have query complexity $\Omega(N)$.

2. Let m be an integer. Construct a permutation matrix whose spectrum (understood here as the set of eigenvalues **without** counting multiplicities) is equal to $\left\{ e^{2\pi i \frac{p}{q}} \mid p \in \mathbb{Z},\ q \in \{1, 2, \ldots, m\} \right\}$.

3. Let us call a Boolean string $a_1, \ldots, a_N$ *mid-Western* if there exists at least one index C such that $a_C = a_{C+1} = 1$ while $a_M = 0$ for any other M with $|C - M| \leq \sqrt{N}$. Let $MW_N(X_1, \ldots, X_N)$ be the characteristic function of the set of all mid-Western strings.

   Determine its sensitivity and block-sensitivity within a multiplicative constant.

4. Let $\ell \leq N$. Consider the partial function

$$F(X_1, \ldots, X_N) \stackrel{\text{def}}{=} \begin{cases} 0 & \text{if } \sum_{x=1}^{N} X_x = 0 \\ 1 & \text{if } \sum_{x=1}^{N} X_x = \ell \\ \text{undefined} & \text{otherwise.} \end{cases}$$

Prove that $Q_2(F) \geq \Omega\left(\sqrt{\frac{N}{\ell}}\right)$.

5. (a) Prove that $\lambda_1(G) \geq \Delta(G)^{1/2}$ for an arbitrary graph G.
   (b) for every $\Delta \geq 0$, give an example of a graph with $\Delta(G) = \Delta$ for which this bound is tight.