Quantum Computing

Instructor: Alexander Razborov, University of Chicago.
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~ razborov/teaching/winter20.html

Winter Quarter, 2020

You may work together on solving homework problems, but please put all the names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Shopping for solutions on the Internet is strongly discouraged but if you do it anyway (or just unintentionally came across it), you must completely understand the proof, explain it in your own words and include the URL.

Homework 3, due March 11

1. Prove or disprove that

$$QC_2\left(\bigwedge_{w=1}^{N}(X_w \oplus Y_w)\right) \geq \Omega(\sqrt{N}).$$

2. Determine the minimal and maximal possible value of the quantity

$$\frac{||H||_{\mathrm{tr}}^2}{||H^2||_{\mathrm{tr}}},$$

where H runs over all Hermitian matrices of size $N \times N$.

3. What is the minimal k for which the completely depolarized state $\frac{1}{N}I_N$ in an N-dimensional Hilbert space can be represented as a mixture of the form $\{(p_1, |\phi_1>), (p_2, |\phi_2>), \ldots, (p_k, |\phi_k>)\}$?

4. Alice picks a mixed state ρ in a 1-qubit space, after that Bob picks another mixed state σ, and then Alice pays Bob $D(\rho, \sigma)$ dollars. Determine the value of this game, the best strategy for Alice and describe the set of best responses for Bob.

5. A one-qubit random quantum circuit consists of a single unitary gate picked uniformly at random from the four Pauli matrices $\{I, \sigma_x, \sigma_y, \sigma_z\}$. Give a (very!) explicit description of the superoperator computed by this circuit.