Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 1, due October 14

1. Find integers x, y such that
$$1773x + 1865y = 1.$$

2. Similarly to the case $k = 2$, define the *greatest common divisor* $\gcd(a_1, \ldots, a_k)$ of the integers $a_1, \ldots, a_k$ as the largest integer d such that $d|a_1, \ldots, d|a_k$.

   Prove that if $\gcd(a, b, c) = 1$ then there exist integers x, y, z such that
$$ax + by + cz = 1.$$

3. Prove geometrically (that is, with a single picture and without a single English word) that
$$1 + 3 + 5 + \ldots + (2n - 1) = n^2.$$

4. $2n$ football players each of whom has an odd number of friends among the others are distributed into two teams. A player is *happy* if he has more friends on the other side than on his own, and the distribution is *stable* if everyone is happy. The *cut number* of a distribution is the number of pairs of friends belonging to the opposite teams.

   Give an example of two different **stable** distributions of the same set of players that have different cut numbers.

5. ([Rosen, Sct. 4.1, Exc. 4]) Prove that

$$1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2.$$