# Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn09.html

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   Prove all of your answers.  Unless otherwise stated, you may
use any method.  The choice of the proof method will not affect
your grade but if we get some particularly elegant and/or unexpected
proofs, we can do them in the class.
   If you work with others put their names clearly at the top of
the assignment.  Everyone must turn in their own independently written
solutions.  Homework is due at the beginning of class.
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## Homework 2, due October 21

1. Give a closed form expression for the number of ordered pairs $\langle A, B \rangle$, where $A, B \subseteq [n]$ are such that $A \cap B = \emptyset$.

2. Prove that a function $f : X \longrightarrow Y$ is injective if and only if for any finite set $Z$ and any function $g : X \longrightarrow Z$ there exists a function $h : Y \longrightarrow Z$ such that $g = h \circ f$.

3. Give a combinatorial bijection between the collection of odd subsets (subsets of odd size) of $[n]$ and the collection of even subsets (subsets of even size) of $[n]$.

   NOTE. The case when $n$ is odd was done in the class.

4. How many integers between 1 and 2009 are divisible by a non-trivial cube $p^3$, $p > 1$?

5. Prove that
$$\sum_{k=0}^{n} k \binom{n}{k}^2 = \frac{n}{2} \binom{2n}{n}.$$