Discrete Mathematics

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Prove all of your answers. Unless otherwise stated, you may use any method. The choice of the proof method will not affect your grade but if we get some particularly elegant and/or unexpected proofs, we can do them in the class.

If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.

Homework 3, due October 28

1. Prove that every prime number p can be represented in the form $\binom{n}{k}$ in only two ways: $p = \binom{p}{1} = \binom{p}{p-1}$.

2. Prove that the number of partitions of n using *at most* k numbers is equal to the number of partitions of $n+k$ using *exactly* k numbers.

3. Let $m = \frac{n^2+n+4}{2}$, and assume that $S_1, S_2, \ldots, S_m \subseteq [n]$ are pairwise distinct. Prove that there exist $1 \leq i < j \leq m$ such that $|S_i \cap S_j| \geq 2$.

4. Citizens of a certain town have formed 17 different committees. Two committees A, B are *comparable* if either every member of A is also a member of B or, vice versa, every member of B is also a member of A. Prove that either there exist five committees that are mutually comparable to each other or there exist five committees that are mutually incomparable.

5. Prove that upon throwing 10 dice, the probability to get a total of 25 is equal to the probability to get a total of 45.