Discrete Mathematics

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Prove all of your answers. Unless otherwise stated, you may
use any method. The choice of the proof method will not affect
your grade but if we get some particularly elegant and/or unexpected
proofs, we can do them in the class.

If you work with others put their names clearly at the top of
the assignment. Everyone must turn in their own independently written
solutions. Homework is due at the beginning of class.

Homework 4, due November 4

1. Calculate the probability that in the game of preference (32 cards dealt
   among three players, 10 cards each, and 2 cards go to the widow) every
   player gets precisely one ace.

2. Give an example of three events in the same sample space such that
   every pair of these events is independent, but all three events are not
   mutually independent.

3. Our class is attended by 26 students, and its graduate version the next
   door is attended by 17 students. Every student in every of the two
   classes is present with probability 90% independently of the others.

   Looking for you, your friend opens one of the two doors at random and
   sees 17 students in the room. What is the probability that he opened
   the right door?

4. Let $X_1, \ldots, X_6$ be independent unbiased Bernoulli random variables
   (that is, $p(X_i = 0) = p(X_i = 1) = 1/2$). Calculate the conditional
   probability $p(X_1 + X_2 + X_3 + X_4 = 2 | X_3 + X_4 + X_5 + X_6 = 1)$.