Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn09.html

Autumn Quarter, 2009

```
   Prove all of your answers.  Unless otherwise stated, you may
use any method.  The choice of the proof method will not affect
your grade but if we get some particularly elegant and/or unexpected
proofs, we can do them in the class.
   If you work with others put their names clearly at the top of
the assignment.  Everyone must turn in their own independently written
solutions.  Homework is due at the beginning of class.
```

Homework 5, due November 11

1. You roll a pair of fair dice. Compute the expectation of the *maximum* of the two results.

2. Prove that for any random variables X, Y on the same sample space (*not* necessarily independent), $V(X+Y) \leq 2(V(X)+V(Y))$.

3. The *mean deviation* $MD(X)$ of a random variable X is defined as $E(|X-c|)$, where $c = E(X)$ is the expectation of X (we briefly discussed this notion in the class).

   (a) Prove that for any two random variables X and Y on the same sample space, $MD(X+Y) \leq MD(X) + MD(Y)$.

   (b) Prove that if X and Y are additionally known to be independent, then this inequality is *always* strict, unless one of the variables X, Y is trivial (that is, takes one fixed value with probability 1).

4. Define a random graph $G(n, p)$ on the set of n vertices as follows: for each pair of vertices u, v, $\{u, v\}$ is declared to be an edge with probability p independently of all others[1]. What is the expected number of unordered triples (u, v, w) of pairwise different vertices such that all three edges $\{u, v\}, \{v, w\}, \{u, w\}$ are present in $G(n, p)$?

5. Draw all pairwise different (that is, non-isomorphic) undirected simple graphs on 4 vertices. No proof is required for this particular exercise.

[1] This model is often called *Erdös-Renyi model*.