Discrete Mathematics

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Prove all of your answers. Unless otherwise stated, you may use any method. The choice of the proof method will not affect your grade but if we get some particularly elegant and/or unexpected proofs, we can do them in the class.

If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.

Homework 7, due November 30

1. Let X be a *non-negative* random variable.
   (a) Prove that $E(X) \geq \sum_{k=1}^{\infty} P(X \geq k)$.
   (b) Prove that equality holds if and only if X takes on only integer values.

2. Consider the graph G_n with the vertex set $\{0,1,2\}^n$ in which two vertices are connected if and only if they differ in precisely one coordinate. Prove that $\chi(G_n) = 3$.

3. Recall that the *strong direct product* $G_1 \times G_2$ of two graphs G_1 and G_2 is given by $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ and

   $$E(G_1 \times G_2) = \{\, \{(u_1,u_2),(v_1,v_2)\} \mid (u_1,u_2) \neq (v_1,v_2) \;\&\; \forall i = 1,2 (u_i = v_i \vee (u_i,v_i) \in E(G_i)) \,\}\,.$$

   Prove that $\alpha(C_5 \times C_5) = 5$.

4. Prove that the Petersen graph does not have a Hamiltonian cycle.

5. Prove that the Petersen graph does not contain a subgraph that is a subdivision of K_5.