Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 1, due October 13

1. Find integers x, y such that
$$1914x + 1939y = 1.$$

2. Similarly to the case $k = 2$, define the *greatest common divisor* $\gcd(a_1, \ldots, a_k)$ of the integers $a_1, \ldots, a_k$ as the largest integer d such that $d|a_1, \ldots, d|a_k$.

   Prove or disprove the following: if a, b, c are integers such that $\gcd(a, b) > 1$, $\gcd(b, c) > 1$ and $\gcd(a, c) > 1$ then $\gcd(a, b, c) > 1$.

3. We are given a party of n ladies and n gentlemen. Suppose that for some integer k, $0 < k \leq n$ there exists a group of k ladies that altogether know at most $(k - 1)$ gentlemen. Prove that there exists a (possibly different) integer ℓ, $0 < \ell \leq n$ and a group of ℓ gentlemen that altogether know at most $(\ell - 1)$ ladies.

4. Let k be a fixed integer. Prove that $1^k + 2^k + \ldots + n^k = \Theta(n^{k+1})$. `Note.` O`-part of this exercise is in Rosen's book, but I want you to do both parts.`

5. Assume that $f, g : \mathbb{N} \longrightarrow \mathbb{R}$ are *strictly positive* functions (that is, $f(n) > 0$ and $g(n) > 0$ for all n) and that $f(n) \leq O(g(n))$. Prove that there exists a constant $C > 0$ such that $f(n) \leq Cg(n)$ holds *for any* n.