Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 3, due October 27

1. A *grid graph* is a graph with vertex set equal to the coordinates of integer points in some block of the plane: $\{(x,y) \mid a < x < b \ \& \ c < y < d\}$ ($a < b$ and $c < d$ are integers) and edge set equal to all pairs of neighboring (vertical and horizontal) lattice points.

   What is the chromatic number of any grid graph?

2. Let G be a planar graph with v vertices, e edges, r regions and k connected components. Show that $r = e - v + k + 1$. `Note. Euler's formula [Rosen, page 660] corresponds to the case` $k = 1$`; this exercise asks you to generalize it.`

3. Prove that $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n. `Hint. Use induction.`

4. Give a closed form expression for the number of ordered pairs $\langle A, B \rangle$, where $A, B \subseteq [n]$ are such that $A \subseteq B$.

5. Prove that a function $f : X \longrightarrow Y$ is surjective if and only if for any finite set Z and any function $g : Z \longrightarrow Y$ there exists a function $h : Z \longrightarrow X$ such that $g = f \circ h$.