Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 4, due November 3

1. An integer is *square-free* if it is not divisible by p^2 for any $p > 1$. Prove that the number of odd square-free integers between 1 and 10^6 equals the number of even square-free integers between 1 and $2 \cdot 10^6$. `Hint. For this exercise, as well as for # 2 and 5 below you have to know the prime factorization theorem, see [Rosen, Thm. 1 on page 211].`

2. An integer is *cube-free* if it is not divisible by p^3 for any $p > 1$. How many cube-free integers are there between 1 and 27100? `Note. In order to qualify for the full credit, the answer must be an integer.`

3. How many ways are there to choose five cards out of a standard deck of 52 cards in such a way that all four suits are represented in the selection? `Large factorials (`$n!$` for `$n \geq 7$`) may be left unexpanded here.`

4. Prove that
$$\sum_{k=0}^{n} k \binom{n}{k} = n2^{n-1}.$$
`Hint. Try to find a bijective proof.`

5. Let p be a prime. What is the smallest integer $n \geq p$ for which $\binom{n}{p}$ is divisible by p?