Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class. Large factorials` $n!$ `with` $n \geq 7$ `may be left unexpanded.`

Homework 5, due November 10

1. Prove that
$$S(n,k) = kS(n-1,k) + S(n-1,k-1),$$
where $S(n,k) \stackrel{\text{def}}{=} \frac{1}{k!}\sum_{r=0}^{k}(-1)^r\binom{k}{r}(k-r)^n$ is the number of equivalence relations on n elements with k equivalence classes.

2. We want to rearrange the phrase **COMPUTATIONAL COMPLEXITY** in such a way that the result:
   (a) consists of two words, the first one having 13 letters and the second word having 10 letters;
   (b) every one of the two words has a connected subword "COMP" in it.

   For example, "XICOMPTTAIANY UOLLECOMPT" is a good arrangement, while "XICOMPTTAIANYU OLLECOMPT", "XICOMPCOMPANY UOLLETTAIT" or "XICOMTPTAIANY UOLLECOMPT" are not. How many good arrangements are there?

3. The pool game consists of seven indistinguishable[1] striped balls, seven

[1] for the purpose of this exercise we disregard the numbers actually written on the balls

indistinguishable solid balls, one white ball and one black ball. A *position* is a placement of any number (possibly zero) of these balls into six distinguishable pockets. How many different positions are there in this game?

4. Prove that $P_k(n)$, the number of partitions of n using at most k numbers, is equal to the number of partitions of $n + k$ using *exactly* k numbers.

5. Prove that a connected graph with n vertices has n edges if and only if it contains *precisely* one simple cycle.