Discrete Mathematics

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   Prove all of your answers.  If you work with others put their
names clearly at the top of the assignment.  Everyone must turn
in their own independently written solutions.  Homework is due at
the beginning of class.  Large factorials n! with n ≥ 7 may be
left unexpanded.
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Homework 6, due November 17

1. How many different spanning trees does the graph on Figure 1 contain?

2. Calculate the probability that a random hand consisting of five cards out of a standard deck of 52 cards contains precisely two aces.

3. Let $A, B \subseteq [n]$ be two random subsets of an n-element set picked uniformly and independently of each other. Calculate $p(A \cup B = [n])$.

4. Give an example of three events A, B, C in the same sample space such that every pair of them is independent while $p(A|B \wedge C) \neq p(A)$.
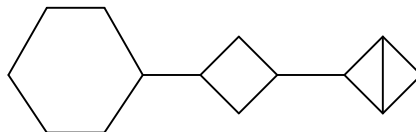

Figure 1: just a nice graph.

5. Every homework assignment in the 27100 class consists of five problems. Alice solves every one of them with probability 90% except for the last one that she solves with probability 30% (independently of each other). Bob solves every one of them with probability 90% except for the first one that he solves with probability 30% (independently of each other). After the homework was handed in, the Professor finds an unsigned sheet that solves all problems but the second, and he knows that it may belong only to Alice or Bob. What is the probability that it belongs to Alice?