Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 2, due October 23

1. Give a closed form expression for the number of ordered pairs $\langle A, B\rangle$, where $A, B \subseteq [n]$ are such that $A \cup B = [n]$.

2. The *Hamming weight* of a binary string is the number of ones in it. Give a combinatorial bijection between the set of all binary stings of length n and even Hamming weight and the set of those that have the same length n and whose Hamming weight is odd.

3. Which of the binomial coefficients $\binom{2013}{500}$ and $\binom{2013}{1500}$ is larger?

4. Prove that there are at least $P(m, n-1) = \frac{m!}{(m-n+1)!}$ surjective functions from $[m]$ to $[n]$.

5. For how many integers n between 1 and $6 \cdot 10^6$ there exists at least one pair of integers (x, y) such that $xn + 60y = 1$?