Discrete Mathematics

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Autumn Quarter, 2013

`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class. Large factorials (`$n!$` for `$n \geq 7$`) may be left unexpanded.`

Homework 3, due October 30

1. How many ways are there to choose nine cards out of a standard deck of 52 cards in such a way that every suit is represented in the selection at least twice?

2. How many solutions in positive integers are there to the equation
$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 2^{20} \cdot 13^{13}?$$

3. Recall that $P_r(n)$ is the number of representations $n = n_1 + \ldots + n_r$ with $n_1 \geq \ldots \geq n_r \geq 1$.

   Which of the two numbers $P_r(n)$ and $P_{r+1}(n+2)$ is larger?

4. Construct an example of a 10-element set A of positive integers in the range [1..1000] such that all 2^{10} *subset sums* $\sum_{a\in S} a$ (S ranges over all subsets of A) are pairwise different.

5. A set $\{a_1, \ldots, a_n\}$ of positive integers is *nice* iff there are no non-trivial (i.e., those in which at least one component is different from 0) solutions to the equation
$$a_1x_1 + \ldots + a_nx_n = 0$$

with $x_1, \ldots, x_n \in \{-1, 0, 1\}$. Prove that any nice set with n elements necessarily contains at least one element that is $\geq \frac{2^n}{n}$.