

Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu

Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn13.html

Autumn Quarter, 2013

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class. Large factorials ($n!$ for $n \geq 7$) may be left unexpanded.

Homework 4, due November 7

1. You throw a fair die 2013 times. What is the probability that the total is divisible by three?
Note. In order to qualify for full credit, the answer must be a rational.
2. Alice and Bob are dealt five cards each from the same 52-cards deck. Calculate the probability that Alice gets a *flush* (five cards of the same suit) and Bob gets four of a kind. Are these two events independent?
3. Our class has 45 students in it, and its graduate version the next door has an enrollment of 30. In our class, every student attends any particular lecture with probability 70% independently of other students, and in Professor Babai's class two thirds of all lectures are attended by everyone (and with probability $1/3$ someone is missing).
Looking for you, your friend opens one of the two doors at random and sees 30 students in the room. What is the probability that he opened the right door?
4. Let A, B, C be events in the same sample space such that every pair of them is independent and such that $p(A|B \wedge C) = p(A)$. Prove that $p(B|A \vee C) = p(B)$.

5. You have one chance out of a million to win jackpot on a slot machine, and you play it five million times in a row. Give a good estimate of the chance that you will collect *exactly* five jackpot prizes.