Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/autumn13.html

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 6, due November 22

1. Prove that if $d_1, \ldots, d_n$ is the degree sequence of a simple graph then the following two sequences also have this property:
   (a) $d_1, d_2+1, d_3, d_4, d_5, d_6, d_7+1, d_8, 3, 1$ $(n=8)$;
   (b) $n-1-d_1, n-1-d_2, \ldots, n-1-d_n$.
2. Let $a, b \geq 2$ be integers. Consider the graph $G = (V, E)$, where $V = \{0, 1, \ldots, a-1\}$ and $E = \{(x, x+b \mod a) \mid x \in V\}$. Prove that G has precisely $gcd(a, b)$ connected components.
3. How many $(s-t)$ paths of length five does the graph on Figure 1 contain?

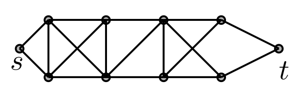

Figure 1: Yet another graph.

4. Define a random graph $G(n, p)$ on the set of n vertices as follows: for each pair of vertices u, v, $\{u, v\}$ is declared to be an edge with probability p independently of all others[1]. What is the expected number of cycles C_4 in this graph? For the purpose of this exercise, two cycles that are obtained from each other by rotations and reflections are considered the same, but the cycles need not necessarily be *induced* (i.e., may contain diagonals).

5. Prove that a connected graph with n vertices has *exactly* n edges if and only if it contains *precisely* one simple cycle.

[1] This model is often called *Erdös-Renyi model*.