Discrete Mathematics

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Autumn Quarter, 2013

`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class. All graphs are simple.`

Homework 7, due December 4

1. Give an example of two graphs that have the same degree sequence, the same number of simple cycles C_ℓ for every particular ℓ and the same number of cliques K_r for every particular r, but at the same time are non-isomorphic.

2. A *giraffic tree* is a rooted tree in which the root has degree one. Prove that there are precisely $n(n-1)^{n-2}$ *labeled* giraffic trees on n vertices $\{1, 2, \ldots, n\}$.

3. Consider the graph G_n with the vertex set $\{0, 1, 2, \ldots, 2012\}^n$ in which two vertices are connected if and only if they differ in precisely one coordinate. Prove that $\chi(G_n) = 2013$.

   `Hint. Solve first the simpler case when 2013 is replaced with 2, and try to generalize this solution.`

4. A graph is *cubic* if all its vertices have degree 3. Prove that if G is a cubic graph on 2012 vertices then $\alpha(G) \geq 503$.

5. Let G be a graph on 2013 vertices with $\alpha(G) = 1007$. Prove that G contains at most 1007 cliques of size 1007.