Computability and Complexity Theory

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`You are encouraged to work on solving homework problems with other students, but please put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 3, due March 7

1. Let $f : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$ be a poly-time computable function such that for all x, $f(x, 0) = 0$ and $f(x, x) = 2012$. Prove that there exists another poly-time computable function $g(x)$ such that $f(x, g(x)) \leq 1006$ and $f(x, g(x) + 1) \geq 1006$ for all x.

   `Note.` Numbers are represented in binary.

2. A *perfect matching* in a graph with $2n$ vertices is a set of n edges that do not have any vertices in common. PERFECT MATCHING is the language that consists of all graphs with even number of vertices that possess at least one perfect matching.

   Construct an *explicit* and *direct* (that is, bypassing the Cook-Levin theorem) poly-time Karp reduction from PERFECT MATCHING to SATISFIABILITY.

3. Prove that there exist two sets A and B such that A is poly-time Cook reducible to B (that is, $A \in \mathsf{P}^B$), but A is not poly-time Karp reducible to B.

4. The complexity class $\oplus\mathsf{P}$ consists of languages L for which there exists a poly-time nondeterministic Turing machine M such that $x \in L$ iff the number of accepting paths of M on input x is odd.

(a) Prove that there exists an oracle A such that $\mathsf{P}^A = (\oplus\mathsf{P})^A$ and that there exists an oracle B such that $\mathsf{P}^B \neq (\oplus\mathsf{P})^B$.

(b) Prove that B can be actually chosen in such a way that the stronger statement $(\oplus\mathsf{P})^B \not\subseteq \mathsf{NP}^B$ holds.