Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/winter16.html

Winter Quarter, 2016

`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 1, due January 20

1. Find integers x, y such that

$$1941x + 2016y = 6.$$

2. The *least common multiple* $\mathrm{lcm}(a, b)$ is defined as the smallest positive integer that is divisible by both a and b.

   Prove the identity

$$\mathrm{lcm}(\gcd(a, b), c) = \gcd(\mathrm{lcm}(a, c), \mathrm{lcm}(b, c)).$$

3. $2n$ football players each of whom has an odd number of antagonists among the others are distributed into two teams, possibly of different sizes. A player is *happy* if he has more antagonists on the other side than on his own, and the distribution is *stable* if everyone is happy. The *cut number* of a distribution is the number of pairs of antagonists belonging to the opposite teams.

   Give an example of two different **stable** distributions of the same set of players that have different cut numbers.

4. Compute the sum

$$2 + 6 + 10 + \ldots + (4n - 2)$$

as a closed form expression.

5. Give a closed form expression for the number of ordered pairs $\langle A, B \rangle$, where $A, B \subseteq [n]$ are such that $|A \cap B| = 1$.