Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class.`

Homework 2, due January 27

1. Which of the following four statements are always true? Prove those and give counterexamples to all others.
   (a) If the composition $f \circ g$ of two functions f and g is injective then f is injective.
   (b) If $f \circ g$ is injective then g is injective.
   (c) If $f \circ g$ is surjective then f is surjective.
   (d) If $f \circ g$ is surjective then g is surjective.

2. Prove that there are at least $P(m, n-1) = \frac{m!}{(m-n+1)!}$ surjective functions from $[m]$ to $[n]$.

3. Prove the identity

$$\sum_{k=2}^{n} \binom{k}{2}\binom{n}{k} = \binom{n}{2} 2^{n-2}.$$

   `Hint: even if, as always, any legitimate proof will bring you full credit, I recommend to look for a combinatorial proof. That is, count the cardinality of the same set in two different ways.`

4. How many integers between 1 and 2016 are divisible by a non-trivial cube p^3, $p > 1$?