Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class. Factorials` $n!$ `with` $n \geq 7$ `may be left unexpanded.`

Homework 3, due February 3

1. What is the coefficient in front of x^{271} in the polynomial $(x^5 + x^2)^{100}$?
2. How many different strings can be made from the letters in CHICAGOLAND, using all letters, and such that no two vowels are adjacent to each other?
3. How many solutions are there to the equation $x_1x_2x_3x_4 = 2016$, where x_is are integers (not necessarily positive)? Solutions obtained from each other by permuting their components are considered different.

   `Hint. For this exercise you will most likely have to know the prime factorization theorem [Rozen, Theorem 1 in Section 4.3].`
4. Prove that $P_r(n)$, the number of partitions of the integer n into at most r positive integers, is equal to the number of partitions of $n + r$ into *exactly* r positive integers.
5. Let n be even. A set $\{a_1, \dots, a_n\}$ consisting of positive integers is *good* if for every two different disjoint subsets $S, T \subseteq [n]$ of the same cardinality we have $\sum_{i \in S} a_i \neq \sum_{i \in T} a_i$. Prove that any good set A with n elements must necessarily contain an element that is greater or equal than $\frac{2}{n}\binom{n}{n/2}$.