Discrete Mathematics

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`Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class (paper submission) or 11:59pm (PDF generated from a (La)TeX source, e-mailed to Leonardo).`

Homework 4, due February 10

1. Prove that upon rolling 2016 dice, the probability to get a total of 2710 is equal to the probability to get a total of 11402.

2. Alice and Bob are dealt five cards each from the same 52-cards deck. Let E be the event that Alice gets a flush (five cards of the same suit) and F be the event that Bob's five cards are of pairwise different kinds. Are E and F independent?

3. Prove or disprove the following. Let A, B, C be three events in the same sample space such that $p(A), p(B), p(C) > 0$ and every pair of these events is independent[1]. Then $p(A \wedge B \wedge C) > 0$.

4. We are given 11 boxes with 10 balls each. Every ball is either red or green, and the ith box ($1 \leq i \leq 11$) has precisely $(i-1)$ red balls in it. Someone picks, uniformly at random, one of those boxes and then picks, also uniformly at random, two balls from it. Calculate the probability that these two balls are of the same color.

[1] Such events are called *pairwise independent*

5. The chances of being born with albinism are estimated as 1 in 1200. Estimate the chance, in the form of a decimal real number, that an island with 10000 inhabitants has precisely 8 albinos. `Note. We assume that all 10000 events of being born with albinism are mutually independent. Also, for this particular problem extensive computer calculations are prohibited: if your plan is to estimate` $\binom{10000}{1200}$`, you must do it by hand!`