Discrete Mathematics

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Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class (paper submission) or 11:59pm (PDF generated from a (La)TeX source, e-mailed to Samira).

Homework 5, due February 17

1. The *mean deviation* $MD(X)$ of a random variable X is defined as $E(|X - c|)$, where $c = E(X)$ is the expectation of X.

   (a) Prove that for any two random variables X and Y on the same sample space, $MD(X + Y) \leq MD(X) + MD(Y)$.

   (b) Prove that if X and Y are additionally known to be independent, then this inequality is *always* strict, unless one of the variables X, Y is trivial (that is, takes one fixed value with probability 1).

2. Let A and B be random subsets of $[n]$ picked uniformly at random from all 2^n subsets, independently of each other. Compute the variance of the random variable $|A \cup B|$.

3. Every one of 56 students in our class independently rolls a standard unbiased die. Prove that the chance to get a total of at least 250 does not exceed 6%.

4. Let $n \geq 2$. Prove that if $d_1, \ldots, d_n$ and $d'_1, \ldots, d'_n$ are degree sequences of simple graphs then $d_1 + 2, d'_1 + 2, \ldots, d_n + 2, d'_n + 2$ is also the degree sequence of a simple graph.