

# Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago  
razborov@cs.uchicago.edu

Course Homepage: [www.cs.uchicago.edu/~razborov/teaching/winter16.html](http://www.cs.uchicago.edu/~razborov/teaching/winter16.html)

Winter Quarter, 2016

Prove all of your answers. If you work with others put their names clearly at the top of the assignment. Everyone must turn in their own independently written solutions. Homework is due at the beginning of class (paper submission) or 11:59pm (PDF generated from a (La)TeX source, e-mailed to Kai).

## Homework 6, due February 24

1. Compute the diameter of the graph  $G$  with  $V(G) = \{0, 1\}^{2016}$  in which two binary strings are connected if and only if they *coincide* in at most one coordinate.
2. Let  $u$  and  $v$  be two adjacent vertices in the cycle  $C_5$ . How many  $(u, v)$ -paths of length 20 there are? Note. For this problem you may use any computational tools, and you need not provide a transcript, but you must clearly state in mathematical terms *what* you are computing.
3. Give an example of two simple graphs that:
  - (a) have the same degree sequences;
  - (b) for any given  $r \geq 2$  have the same number of copies of  $K_r$ ;
  - (c) for any given  $\ell \geq 3$  have the same number of induced copies of  $C_\ell$ ,but nonetheless are not isomorphic to each other.
4. Prove that  $\chi(G) \leq 4$ , where  $G$  is the graph from Exercise 1.