Discrete Mathematics

Instructor: Alexander Razborov, University of Chicago
razborov@cs.uchicago.edu
Course Homepage: www.cs.uchicago.edu/~razborov/teaching/winter16.html

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Prove all of your answers.  If you work with others put their
names clearly at the top of the assignment.  Everyone must turn
in their own independently written solutions.  Homework is due at
the beginning of class (paper submission) or 11:59pm (PDF generated
from a (La)TeX source, e-mailed to Samira).
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Homework 8, due March 9

1. Prove that the Petersen graph does not contain a subgraph homeomorphic to K_5.

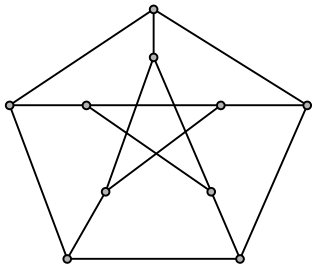

Figure 1: The Petersen graph.

2. Prove that any graph that can be obtained from the Petersen graph by adding one extra edge has a Hamiltonian cycle.

3. Recall that the *cube* Q_n is the simple graph with $V(G_n) = \{0,1\}^n$ in which two vertices are adjacent if and only if they differ in exactly one coordinate.

Prove that the edges of Q_{2016} can be oriented in such a way that there will be exactly 1008 edges going out of any vertex.

4. Prove that a matrix T is stochastic if and only if it can be represented in the form $T = p_1F_1 + \ldots + p_mF_m$, where $p_i \geq 0$, $\sum_{i=1}^{m} p_i = 1$ and F_i are stochastic matrices in which all entries are either 0 or 1.

5. Prove that the Markov chain obtained from the Petersen graph as shown on Figure 2 has a unique stationary distribution and find it.

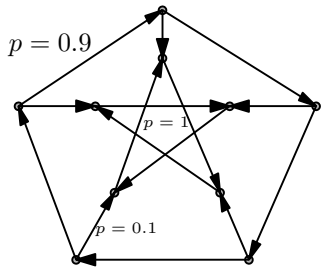

Figure 2: All outer edges carry probability 0.9, all cross-edges carry probability 0.1, and all inner edges carry probability 1.