Computability and Complexity Theory

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You are encouraged to work on solving homework problems with
other students, but please put their names clearly at the top of
the assignment.  Everyone must turn in their own independently written
solutions.  Homework is due at the beginning of class.
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Homework 3, due March 9

1. Construct an *explicit* and *direct* poly-time Karp reduction **from** INDEPENDENT SET **to** SATISFIABILITY *bypassing the Cook-Levin theorem*.

2. Prove that the following problem is in **P**:

   `INPUT.` A position in the game of checkers on an $n \times n$ board with at most 2010 pieces, white moves first.

   `QUESTION.` Does the first player have a winning strategy?

3. Let $t : \mathbb{N} \longrightarrow \mathbb{N}$ be any poly-time computable (in binary representation) function of superpolynomial growth, i.e., for any polynomial $p(n)$ there exists n_0 such that $\forall N \geq n_0(t(N) \geq p(N))$. Let L be any language. Prove that if L is poly-time Cook (!) reducible to $\left\{ (x, 1^{t(|x|)}) \mid x \in L \right\}$ then $L \in \mathbf{P}$.

4. A *perfect square* machine is a non-deterministic machine that outputs YES if and only if the number of computational paths leading to the state q_{accept} is a perfect square (that is, one of 0,1,4,9,16,25...). **PS** is the class of all languages recognizable by poly-time perfect square machines.

Prove that there exist two oracles A and B such that $\mathbf{NP}^A = \mathbf{PS}^A$ and $\mathbf{NP}^B \neq \mathbf{PS}^B$.

5. (Arora, Barak, Excercise 5.9) Instead of part a), prove that $\mathbf{DP} \subseteq \Pi_2^p$ (a) is readily implied by this and part b)).