

0.1 Functional iteration for systems, Chapter 3, Section 2

We just need to interpret the notation: g now maps R^n to itself, and fixed point iteration seeks to find a fixed point

$$\alpha = g(\alpha) \tag{1}$$

where now $\alpha \in R^n$. Fixed point iteration

$$x^\nu = g(x^{\nu-1}) \tag{2}$$

still has the property that, if it converges, it converges to a fixed point (1) (assuming only that g is continuous).

The rest of the story is similar, except that we need some higher-dimensional calculus to figure out when and how fast it will converge. Basically, if g is Lipschitz continuous with constant $\lambda < 1$, that is,

$$\|g(x) - g(y)\| \leq \lambda \|x - y\| \tag{3}$$

for some norm $\|\cdot\|$ on R^n , then convergence will happen if we start close enough to α .

0.1.1 Explicit methods, Chapter 3, Section 3.1

Not all of the one-dimensional methods generalize to n -dimensions. Suppose we want to solve $f(x) = 0$ where now f maps R^n to itself. The chord method becomes

$$x^{\nu+1} = x^\nu - Af(x^\nu). \tag{4}$$

where A is a matrix.

Newton's method takes $A = J_f(x^\nu)^{-1}$; that is, we solve

$$J_f(x^\nu)(x^{\nu+1} - x^\nu) = -Af(x^\nu). \tag{5}$$

0.1.2 Convergence, Chapter 3, Section 3.2

Convergence of Newton's method is again quadratic. The "abc" condition of Theorem 3 gives global conditions on convergence. Note that if $J_f(x^\nu)$ is nearly singular at any point, then the change $x^{\nu+1} - x^\nu$ can be huge. This occurs even in one dimension: consider $f(x) = \cos x$, and start Newton near $x = 0$.

0.1.3 Special methods for polynomials, Chapter 3, Section 4

Horner's rule for computing $p(\xi)$ yields as a byproduct a decomposition $p(x) = (x - \xi)q + r$ where $r = p(\xi)$ and the degree of q is one less than the degree of p . Differentiating, we find $p'(\xi) = q(\xi)$, so we have both of the things needed for Newton's method.

Please read the section on Bernoulli's Method (page 128 and ff.) to prepare to do homework problem (30) below (page 133, number 3).

0.1.4 Homework

- (27) page 123, number 3
- (28) page 123, number 4
- (29) page 133, number 1
- (30) page 133, number 3

References