- The sets J_k and R_k are defined by

$$J_k = [2^k, 2^{k+1}) \cap \mathbb{N} \qquad \text{and} \qquad R_k = \{2^k l \mid l \in \mathbb{N}, l \text{ odd}\}$$

- ρ_k and d_k are defined by

$$\rho_k(A) = \frac{|A \cap [0,k)|}{k} \qquad \text{and} \qquad d_k(A) = \frac{|A \cap J_k|}{2^k}$$

- The **upper density** $\overline{\rho}(A)$, and $d(A)$, are defined by

$$\overline{\rho}(A) = \limsup_{k\to\infty} \rho_k(A) \qquad \text{and} \qquad \overline{d}(A) = \limsup_{k\to\infty} d(A)$$

  A is **sparse** if $\overline{\rho}(A) = 0$ and **dense** if its complement is sparse.

- **Theorem 1.** $\overline{\rho}(A) = 0$ if and only if $\overline{d}(A) = 0$.

- A **coarse description** of a function f is a total function g such that the set $\{n \mid f(n) = g(n)\}$ is dense. A set is **coarsely computable** if its characteristic function has a computable coarse description.

- A **generic description** of a function f is a partial function g such that $f(n)\downarrow$ implies $g(n)\downarrow = f(n)$ and such that the domain of f is dense. A set is **generically computable** if its characteristic function has a partial computable generic description.

- A **dense description** of a function f is a partial function g such that the set $\{n \mid f(n)\downarrow = g(n)\downarrow\}$ is dense. A set is **densely computable** if its characteristic function has a partial computable dense description.

- An **effective dense description** of a function f is a total function g whose image is $\mathbb{N} \cup \{\square\}$ (where $\square$ is a special symbol which means "don't know"), that $g(n) \neq \square$ implies $g(n) = f(n)$, and that the set $\{n \mid g(n) \neq \square\}$ is dense. A set is **effectively densely computable** if its characteristic function has a computable effective dense description.

- For a set A, the sets $\mathcal{R}(A)$ and $\tilde{\mathcal{R}}(A)$ are defined by

$$\mathcal{R}(A) = \bigcup_{k\in A} R_k \qquad \text{and} \qquad \tilde{\mathcal{R}}(A) = \bigcup_{k\in A} J_k.$$

- An **enumeration operator** W is a c.e. collection of pairs (F, k), where $k \in \mathbb{N}$ and F is a finite subset of $\mathbb{N}$.

- Given an enumeration operator W and a set A, we define W^A by

$$W^A = \{k \mid (F,k) \subseteq W \text{ for some } F \subseteq A\}.$$

- A set A is **uniformly generically reducible** to a set B if there exists an enumeration operator W such that, for all generic descriptions f of A, the set $W^{\text{graph}\, f}$ is the graph of a generic description of B.

- A set A is **uniformly densely reducible** to a set B if there exists an enumeration operator W such that, for all dense descriptions f of A, the set $W^{\text{graph}\, f}$ is the graph of a dense description of B.

- A set A is **uniformly coarsely reducible** to a set B if there exists a Turing operator Φ such that, for all coarse descriptions C of A, the function Φ^C is a coarse description of B.

- A set A is **uniformly effectively densely reducible** to a set B if there exists a Turing operator Φ such that, for all effectively dense descriptions C of A, the function Φ^C is an effectively dense description of B.