Fundamental Analysis of Full-Duplex Gains in Wireless Networks

Shu Wang, Vignesh Venkateswaran, and Xinyu Zhang

Abstract—Full-duplex radio technology is becoming mature and holds potential to boost the spectrum efficiency of a point-to-point wireless link. However, a fundamental understanding is still lacking, with respect to its advantages over half-duplex in multi-cell wireless networks with contending links. In this paper, we establish a spatial stochastic framework to analyze the mean network throughput gain from full duplex, and pinpoint the key factors that determine the gain. Our framework extends classical stochastic geometry analysis with a new tool set, which allows us to model a tradeoff between the benefit from concurrent full-duplex transmissions and the loss of spatial reuse, particularly for CSMA-based transmitters with random backoff. We analytically derive closed-form expressions for the full-duplex gain as a function of link distance, interference range, network density, and carrier sensing schemes. It can be easily applied to guide the deployment choices in the early stage of network planning.

Index Terms—Full duplex, capacity, stochastic geometry.

I. INTRODUCTION

R ECENT advances in radio hardware and signal processing are pushing full-duplex wireless communications close to commercialization [1]. However, existing work mostly focused on full-duplex PHY-layer implementation [2], [3] or MAC protocols [4], [5] that extend 802.11 CSMA/CA. Unlike half-duplex wireless networks whose asymptotics have been investigated extensively [6], the fundamental network-capacity implications of full-duplex remain largely unexplored.

In distributed wireless networks, contending nodes’ transmissions need to be separated in time, frequency, and/or space to avoid excessive interference. Whereas full-duplex allows a pair of nodes to co-locate their transmissions in the same time slot and frequency band, their spatial interference footprint is heavier than a half-duplex pair. An accurate characterization of this trade-off can lead to a fundamental understanding of the full-duplex network capacity and the achievable gain, thus guiding the practical protocol design and network deployment.

The objective of this work is to provide an analytical framework allowing one to access the key properties of full-duplex wireless networks running random access MAC protocols. The insights we seek include, e.g., what is the expected network throughput (gain) when using full-duplex radios compared with half-duplex ones? What are the key factors that determine the gain and how to engineer such design knobs to maximize full-duplex’s potential?

For such an analytical model, the main challenge lies in a need to take into account interference, random contention, and the resulting spatial reuse among contending links. Such factors, of course, are topology dependent. One cannot traverse the enormous number of possible configurations, but must instead consider a statistical spatial model for the node locations, and extracts insights from there.

Following this principle, we assume certain statistical distribution of AP/client locations, and derive spatial averages of critical network quantities, e.g., interference and spatial density of successful transmitters. Such a spatial averaging technique, widely referred to as stochastic geometry [7], has been used in a variety of wireless network examples, like ad-hoc networks, in order to perform average-case analysis of network throughput, by modeling the interference experienced by nodes under a random access MAC protocol.

It is, however, non-trivial to apply the classical stochastic geometry model to full-duplex networks, because of two new barriers. First, existing stochastic geometry analysis [8], [9] uses a hard-core point process (HCPP) to model the distribution of winning transmitters. The contention region of a point in HCPP is defined by a unit disc containing no other points. With full-duplex, the spatial footprint of two neighboring transmitters can become correlated, which can no longer be handled by conventional stochastic geometry models. Second, existing models only focus on winning transmitters after CSMA/CA contention, but ignore the receiver which itself has an exclusive region and is vulnerable to artifacts of carrier sensing such as hidden terminals. Such artifacts are critical to spatial reuse and to the real gain from full-duplex.

In light of the above challenges, we propose a new stochastic framework that can analyze the average spatial footprint of a typical full-duplex pair, as well as the spatial distribution of full-duplex pairs that win contention. Our approach leads to closed-form expressions for the average throughput of full-duplex networks with Poisson-bipolar distributed links. It also gives closed-form analysis of half-duplex throughput under carrier sensing artifacts, e.g., hidden/exposed terminals. Consequently, we can derive the full-duplex throughput gain under a variety of topological parameters and protocol imperfections.
Our analysis starts with full-duplex ALOHA, the basic random access protocol where each node transmits with a fixed probability and without carrier sensing. Then, we extend the model to the more sophisticated CSMA. We find that the most critical factor that determines full-duplex gain is the mean link distance $d$ relative to the carrier sensing range. A smaller $d$ amplifies full-duplex gain since, intuitively, it reduces the interference footprint of a full-duplex link. For a fixed $d$, full-duplex gain tends to be larger in a very sparse deployment of APs, yet the gain saturates quickly as density increases. Surprisingly, we found that full-duplex ALOHA has even lower throughput than half-duplex ALOHA as $n$ and $d$ go beyond a certain level. The fundamental reason again lies in much heavier interference footprint.

More interestingly, we found a major contributing factor to full-duplex gain in CSMA networks lies in full-duplex nodes’ capability to implicitly remove hidden/exposed terminals. Thus, the full-duplex gain tends to be amplified under imperfect carrier sensing.

The rest of this paper is structured as follows. We first present a background on stochastic geometry and our network models in Sec. II. Then we analyze the full-duplex gain under two sets of interference models, in Sec. III and IV. In each interference models, we will start with ALOHA, and then extend to CSMA-like wireless network. This is due to that, in ALOHA, each source independently transmit data whenever it has frames to be transmitted. However, for CSMA protocol, each source need to verifies the channel before any transmission. Sec. V discusses related work and finally, Sec. VI concludes the paper.

II. BACKGROUND AND OVERVIEW OF NETWORK MODELS

In this section, we present the essential models and assumptions underlying our analytical framework.

A. A Primer on Stochastic Geometry and Its Limitations

Stochastic geometry provides average-case analysis of network throughput, wherein the averages are made over a large number of nodes randomly located in the spatial domain.

Recent stochastic geometry models of 802.11 CSMA networks commonly apply a two-step approach [8]. First, nodes are assumed to be deployed following a Poisson Point Process (PPP). Then, the distribution of simultaneously active transmitters after CSMA contention is approximated by a Matern hard core point process (HCPP). Simply put, the HCPP thins the parent PPP and models the winning nodes after random backoff. Second, the interference experienced by a typical winning node is approximated by the interference resulting from a PPP which has the same intensity as the HCPP. Such approximation has been shown to be fairly accurate, mainly because the exact locations of the active transmitters matter less than the number of other active transmitters (interferers) and their relative distances. Given the approximated HCPP, network performance metrics such as transmission success probability (under interference) and throughput can be easily derived.

When applied to modeling the full-duplex gain, existing stochastic geometry models fall short of accuracy from three aspects. (i) They mainly focus on potential transmitters through a homogeneous point process model. The spatial reuse between transmitters and receivers cannot be modeled but is the most critical factor that determines the full-duplex gain [10]. (ii) They assume a unit disk exclusive region around each transmitter, and omit the carrier sensing artifacts, such as exposed and hidden terminals, which again account for the discrepancies in theoretical and practical limit of both half-duplex and full-duplex networks. (iii) They commonly approximate the received signal-to-interference-plus-noise ratio (SINR) using the SINR at the transmitter side, yet whether a transmission succeeds depends SINR at the receiver side (or both sides for full-duplex).

We remedy the above limitations by marrying stochastic geometry with the two interference models proposed in Gupta and Kumar’s seminal work on ad-hoc network capacity analysis [6]. Below we provide more details of our model.

B. Full-Duplex Communication Model

A full-duplex node can simultaneously transmit and receive different packets. State-of-the-art full-duplex radio [3] can isolate the self-interference from transmitted signals to received ones, although perfect elimination is infeasible. Our analysis mainly focuses on the network-level impacts of full-duplex transmissions, assuming perfect full-duplex radio hardware.

When applied to multi-cell wireless LANs, full-duplex links can operate in two modes [2]. Bidirectional transmission mode (Fig. 1(a)) allows a pair of AP-client to transmit packets to each other simultaneously. Cut-through transmission mode (Fig. 1(b)) enables a full-duplex AP to simultaneously serve two clients, one for uplink and the other downlink. When applied to multi-hop networks, it is also referred to as wormhole relaying [10]. We first focus on the former mode, and then prove that the latter results in lower capacity (Sec. III-D.4). In what follows, all references to “full-duplex” means the bi-directional mode, unless noted otherwise.

C. Network Topology Model

We model the locations of transmitters/receivers as some realizations of random point process. Unlike existing CSMA stochastic geometry analysis that commonly focus on Poisson-distributed transmitters (Fig. 2(a)), we model the transmitter and receiver locations using a Poisson bipolar model [11].

For a half-duplex network, transmitters are distributed following a PPP. Each transmitter $T_X$ associates with a receiver $R_X$, located in a direction $\theta$ (Fig. 2(b)), randomly uniformly distributed in $[0, 2\pi)$. We first assume link distance is fixed to $d$, and then generalize the model to random link distance.
The interference power is the cumulative interference from all concurrent transmitters, which still exist outside the transmitter’s carrier sensing range after CSMA contention. We defer the formal mathematical definition to Sec. IV.

III. FULL-DUPLEX GAIN UNDER THE PROTOCOL MODEL

In this section, we describe our stochastic geometry framework that establishes a closed-form analysis of full-duplex gain under the protocol interference model. The analysis derives the spatial throughput of both ALOHA and CSMA-based full-duplex networks. Besides, in order to derive the expected throughput gain from full-duplex, our analysis needs to model carrier sensing artifacts of half-duplex networks, including the hidden/exposed terminals and use of RTS/CTS.

In each case, the analysis follows two major steps: (i) analyze the mean contention region around a typical pair of nodes. (ii) derive the probability of successful transmission for the typical pair that runs the CSMA random backoff, given the Poisson bipolar distributed contending links within its mean contention region. Then we compute and compare the full-duplex spatial throughput with all the half-duplex models to obtain the full-duplex gain in each case.

A. Mean Contention Region (MCR)

We first introduce a novel analytical technique called mean contention region (MCR) that overcomes the aforementioned limitations of classical stochastic geometry models.

Definition: Given a typical link \( L_o \) and a bounded region \( \Omega \subset \mathbb{R}^2 \). We arbitrarily partition \( \Omega \) into \( n \) small regions represented by their areas: \( \Delta \Omega_1, \Delta \Omega_2, \ldots, \Delta \Omega_n \). Let \( \sigma = \max_{1 \leq j \leq n} \Delta \Omega_j \). We randomly select a point \( X_i \) from region \( \Delta \Omega_i \), and define the Mean Contention Region as,

\[
\lim_{\sigma \to 0} \sum_{i=1}^{n} p(X_i) \Delta \Omega_i,
\]

where \( p(X_i) \) is the probability that a transmitter of another link \( L_i \) located at \( X_i \) contends with typical link \( L_o \). If the limit exists and is unique, then we can cast it as \( \int_{\Omega} p(X) d\Omega \). Since \( p(X) \) is a continuous function on a bounded region, the integral exists and is finite.

Intuitively, MCR represents a spatial average of the area within which contenders/interferers may be located. Since the link locations follow a stationary distribution, it suffices to analyze the MCR of a typical link \( L_o \), comprised of a transmitter and a receiver (or two full-duplex bi-directional transmitters).

In half-duplex ALOHA networks, a transmission succeeds when there is no other receiver within the transmitter’s interference range, and no other transmitter within the receiver’s interference range. Essentially, this condition is the same as that in half-duplex CSMA with perfect carrier sensing. Therefore, these two types of networks share the same spatial reuse region, and hence the same MCR. Similarly, full-duplex ALOHA and CSMA networks have the same MCR. However, the eventual probability of a successful transmission differ due to the lack of carrier sensing in ALOHA. We defer the related MCR calculation in Sec. III-A.1 and Sec. III-A.4, and spatial distribution of successful transmissions in Sec. III-B.
For CSMA networks, the definition of contenders/interferers, and the corresponding $p(X)$, depend on not only the duplex mode, but also the carrier sensing. Hence the MCR needs to be analyzed separately, for 4 different categories: half-duplex with perfect carrier sensing; half-duplex with imperfect carrier sensing; half-duplex with RTS/CTS signaling; and full-duplex.

1) Half-Duplex With Perfect Carrier Sensing: Perfect carrier sensing assumes perfect knowledge of contenders: each transmitter is well aware of which receivers it interferes with and which transmitters interfere with its receiver. Therefore, there exist no hidden/exposed terminals and spatial reuse is perfect (Fig. 3(a)). This model is especially useful considering the recent advances in cross-layer implementation that minimizes the impact of hidden [12] and exposed terminals [13]. It is also the basic assumption behind Gupta and Kumar’s protocol model for CSMA networks [6]. The following theorem offers a closed-form characterization of the corresponding MCR. For simplicity of exposition, we only provide the essential steps behind our analysis.

**Theorem 1:** The mean contention region for half-duplex CSMA networks with perfect carrier sensing is given by

$$V_{HP} = \pi R_I^2 + \frac{2}{\pi} \int_{R_I - d}^{R_I + d} (\pi - \theta)\theta dr$$

$$\theta = \arccos\left(\frac{d^2 + r^2 - R_I^2}{2dr}\right).$$

**Proof:** Consider a typical link $L_o$ whose transmitter $T_X$ is located at the origin and its receiver at distance $d$ along the x-axis (Fig. 4(a)).

First, the receiver’s interference range (white area in Fig. 4(a)) should be counted deterministically within MCR (first term on the RHS of Eq. (2)), because any transmitter from other contenting link $L'$ therein will contend with the typical link $L_o$.

Second, consider a contending link $L'$ whose transmitter $T_{X'}$ is within the shaded area in Fig. 4(a). If its receiver $R_{X'}$ is located within the interference range of $T_X$ (red solid arc), then $R_{X'}$ will be interfered. Otherwise (green dashed arc), it needs not contend with the typical link and can transmit concurrently under perfect carrier sensing. Under Poisson bipolar model, the orientation of a receiver w.r.t. its transmitter is uniformly distributed in $[0,2\pi]$. Thus, we can obtain the probability of $R_{X'}$ located in $T_X$’s interference range by calculating the ratio of $\theta$ to $2\pi$. Since this probability $p(X)$ depends on the transmitter’s location, we can integrate the probability throughout the shaded area to obtain the spatial average (second term on the RHS of Eq. (2)).

For any other transmitter outside the above two regions, its receiver $R_{X'}$ can never fall within $T_X$’s interference range, and thus it should not be counted into the MCR.

2) Half-Duplex With Imperfect Carrier Sensing: In the basic 802.11 protocol (Fig. 3(b)), a node defers its transmission whenever it senses a busy channel. This mechanism reduces the risk of collision but often leads to the exposed terminal problem, i.e., some nodes may not interfere a receiver, but are unnecessarily suppressed by the corresponding transmitter. In addition, it suffers from the hidden terminal problem, i.e., other nodes outside the ongoing transmitter’s carrier sensing range but inside the ongoing receiver’s interference range can still cause collisions. We refer to this category of protocol as imperfect carrier sensing, and analyze the MCR as follows.

**Theorem 2:** Under imperfect carrier sensing, the mean contention region for CSMA networks is given by

$$V_{HI} = V_u + 2\int_{R_I}^{R_I+d} (\pi - \theta)\theta dr + d\sqrt{R_I^2 - \frac{d^2}{4}}$$

$$\theta = \arccos\left(\frac{d^2 + r^2 - R_I^2}{2dr}\right).$$

**Proof sketch:** The transmitter $T_X$ suppresses all other transmitters within its interference range which, together with the receiver’s interference range, become a deterministic contention region (the $V_u$ term above, corresponding to the white region in Fig. 4(b)). Spatial average of contention region for the shaded area can be derived in a similar way to Theorem 1.

3) Half-Duplex With RTS/CTS Signaling: An enhanced version of 802.11 uses RTS/CTS to alleviate hidden terminals (Fig. 3(c)). Yet it still bears the exposed terminal problem. Moreover, there may still be hidden terminals outside the CTS transmission range but within the receiver’s interference range. Denote the transmission range as $R_S$, then we can derive the MCR under RTS/CTS signaling as follows.
Theorem 3: The mean contention region for half-duplex network using RTS/CTS is given by

\[
V_{HRC} = \begin{cases} 
V_1 + V_2 + V_3 + 2(V_4 - V_5) & d > R_I - R_S \\
V_1 & d \leq R_I - R_S 
\end{cases}
\]

(7)

\[
V_1 = 2\pi R_1^2 - 2R_2^2 \arccos\left(\frac{d}{2R_1}\right) + d \sqrt{R_1^2 - \frac{d^2}{4}}
\]

(8)

\[
V_2 = \frac{2}{\pi} \int_{R_1}^{R_1+d} (\pi - \gamma_1 - \theta_2)\theta_1 r dr
\]

(9)

\[
V_3 = \frac{2}{\pi} \int_{R_1}^{R_2+d} (\pi - \gamma_2 - \theta_3)\theta_3 r dr
\]

(10)

\[
V_4 = \frac{1}{2\pi} \int_{0}^{d} \int_{\theta_4+\gamma_1}^{\pi-\theta_4-\gamma_1} (\varphi_1 + \varphi_2 + \varphi_3) r dr d\theta
\]

(11)

\[
V_5 = \frac{1}{2\pi} \int_{R_1}^{R_2} \int_{\theta_4}^{\pi-\theta_4-\theta_5} (\varphi_4 + \varphi_5 + \varphi_4) r dr d\theta
\]

(12)

The \(\gamma_1, \gamma_2, \theta_1 \) to \(\theta_5\) and \(\varphi_1 \) to \(\varphi_5\) are intermediate parameters whose detailed expressions are available in Appendix VI-B.

Proof: The detailed proof is in Appendix VI-B.

B. Spatial Density of Successful Transmissions

We now derive the mean transmission density, i.e., average number of successful transmissions per unit area, after the typical pair of nodes contend with peers in the MCR. We start with ALOHA network analysis, which is based on Matérn Type I process. Based on Matérn Type I’s result, we will extend our framework to CSMA network, which is based on Matérn Type II process.

1) Spatial Density of ALOHA Networks: For ALOHA networks (Sec. II-D.1), the distribution of successful transmitters can be modeled as a Matérn Type I process [11] that thins the original PPP distributed based on the medium access probability \(p_m\). After the Matérn thinning and independent medium access, the Palm retaining probability that a point \(x\) of the process is retained equals \(e^{-\mu p_m V}\), where \(\mu\) is the density of the original PPP [11] and \(V\) is the contention region as defined a priori.

(i) For half-duplex ALOHA networks, each transmitter independently decides to access the medium with probability \(p_m\). Therefore, the active transmitters’ distribution still follows a PPP, but with density \(p_m \lambda_p\) (recall that \(\lambda_p\) denotes the original deployment density of all nodes). The contention region for half-duplex ALOHA is the same as \(V_{HP}\) in Theorem 1, since both the deterministic contention region of the receiver’s interference ranges (white area in Fig. 4(a)) and probabilistic contention region of outside area (gray area in Fig. 4(a)) are the same as before. According to the Palm retaining probability mentioned above, the success probability of a typical half-duplex pair is thus given by: \(\exp(-p_m \lambda_p V_{HP})\), and successful density is \(\lambda_{HA} = p_m \lambda_p \exp(-p_m \lambda_p V_{HP})\).

(ii) For full-duplex, we assume the primary and secondary transmitters access the medium simultaneously, which can be realized using MAC protocols as in [14]. The contention region for full duplex ALOHA is the same as \(V_F\) in Theorem 4, since they have the same contention probability for the same location. Based on the Palm retaining probability, the success probability of a typical full-duplex pair is given by: \(\exp(-p_m \lambda_p V_F)\), and successful density is \(\lambda_{FA} = p_m \lambda_p \exp(-p_m \lambda_p V_F)\).

2) Spatial Density of CSMA Networks: The CSMA contention (Sec. II) can be modeled as a Matérn Type II process [15] that thins the original PPP distributed contenders within the MCR. Palm retaining probability calculates the probability of that a point is retained within some locations (116, Ch. 8)). Therefore, given a stationary independently marked PPP of intensity \(\mu\), a point \(x\) of the process having mark \(t\) is retained after Matérn Type II thinning is given by \(e^{-\mu V}\), where \(V\) is the contention area. We adapt this result to our analysis and obtain the density of successful transmissions in the aforementioned 4 cases. Note that existing work widely uses a deterministic unit disk to model \(V\), whereas our MCR model enables a spatial average analysis of the contention region that can model all 4 cases.

Half-Duplex CSMA Models: Let \(\lambda_p\) be the original deployment density of all potential transmitters. For the perfect carrier sensing case, the probability of the typical pair winning contention is \(\exp(-\lambda_p V_{HP})\), where \(V_{HP}\) follows Theorem 1. The mean successful density after contention can be obtain by
Fig. 5. Successful transmission density vs. link distance for (a) half-duplex ALOHA and (b) full-duplex ALOHA networks $R_I = R_C = 100, p_m = 0.6$.

approaches $R_I$, because a longer link-distance is more vulnerable to interference and contention. This is true for both ALOHA and CSMA-based networks. In addition, note that the successful transmission density of the ALOHA networks drops much faster than that of CSMA-based networks when $d$ increases, due to lack of carrier sensing or collision avoidance.

For half-duplex networks, perfect carrier sensing results in higher density than the other two cases. RTS/CTS alleviates hidden-terminals, but the protection sacrifices spatial reuse, and results in similar density as the imperfect carrier sensing. Although the density of full-duplex pairs is similar to that of half-duplex when $d$ is small, it decreases faster, implying that it is more vulnerable to loss of spatial reuse as $d$ increases.

We proceed to analyze the impacts of the successful transmission density on full-duplex gain.

C. Throughput and Full-Duplex Gain

The mean density of successful transmissions can be regarded as spatial-average of the network throughput, which also equals its time-average across stationary topology realizations (Sec. II-C). Therefore, we can derive the full-duplex throughput gain over half-duplex perfect carrier sensing as: $G_{HF} = 2\lambda_F/\lambda_{HF}$, where a multiplier factor 2 is needed since each full-duplex pair supports double link transmission. The gain over other half-duplex cases in CSMA-based networks and ALOHA follows the same derivation.
From the above analytical insights, we can conclude that under the protocol interference model and CSMA contention model, the full-duplex gain is between 140% to 200%, and it largely comes from full-duplex’s capability to overcome the imperfect carrier sensing in half-duplex.

D. Discussion

1) Approximation of Hidden Terminal Scenario: Recall that our analysis of the imperfect half-duplex CSMA and RTS/CTS approximated the hidden terminal scenario. We hereby justify the approximation, starting with a simple 2-link case with imperfect carrier sensing. Consider two link pairs \((T_X_1, R_X_1)\) and \((T_X_2, R_X_2)\) with marks \(t_1\) and \(t_2\), respectively (Fig. 3(b)). Suppose \(T_X_2\) lies in the interference region of \(R_X_1\). When \(t_1 < t_2\), the pair \((T_X_1, R_X_1)\) will be elected to transmit but \((T_X_2, R_X_2)\) will not, because \(R_X_1\) with lower mark lies in the interference region of \(T_X_2\). In reality, as \(T_X_1\) and \(T_X_2\) cannot hear each other, both pairs will transmit. However, \((T_X_1, R_X_1)\) will fail due to \(T_X_2\) interfering \(R_X_1\). In case when \(t_1 > t_2\), \((T_X_2, R_X_2)\) will be elected for transmission and will succeed. In both cases, we end up with one successful transmission, which is consistent with our analysis of \(Y_{HI}\).

Now we establish a theorem to show that it is sufficient to focus on the 2-link case, because the probability that more than one hidden terminals exist within the receiver’s interference range is negligible.

**Theorem 5:** Under imperfect carrier sensing, the probability that more than one hidden terminal exists within the receiver’s interference range can be bounded by,

\[
\mathbb{P}(H_{HI}) \leq 1 - 2 \left( 1 - e^{-\lambda p V_h} \right)
\]

where \(V_h\) denotes the Rx’s interference region excluding its Tx’s interference region.

**Proof:** Hidden terminal happens when (1) \(T_X\) and \(R_X\) are in transmission period, and (2) another active \(T_X'\) falls in \(R_X\)’s interference range but outside \(T_X’\)’s, i.e., \(T_X' \in V_h\). So both \(T_X\) and \(T_X'\) can be considered as retained after contention.

Let \(\Phi\) be the original PPP, \(\tilde{\Phi}\) the process after carrier sensing, and \(\Phi(V)\) the number of points inside any region \(V\) after carrier sensing. Consider a typical Tx located at \(o\), with interference region \(V_o\). The event that \(o\) is retained is \(\tilde{\Phi}(V_o) = 0\), i.e., no other Tx falls within \(V_o\). Similarly, more than one hidden terminal falls within \(V_h\) implies \(\tilde{\Phi}(V_h) \geq 2\). Therefore, we can rewrite the probability as follows:

\[
\mathbb{P}(H_{HI}) = \mathbb{P}(\tilde{\Phi}(V_o) = 0 \cap \tilde{\Phi}(V_h) \geq 2)
\]

\[
= \mathbb{P}(\tilde{\Phi}(V_h) \geq 2) \mathbb{P}(\tilde{\Phi}(V_h) = 0 | \tilde{\Phi}(V_h) \geq 2) \geq 2
\]

\[
\leq \mathbb{P}(\tilde{\Phi}(V_h) \geq 2) \leq 1 - \mathbb{P}(\tilde{\Phi}(V_h) = 0) - \mathbb{P}(\tilde{\Phi}(V_h) = 1)
\]

\[
= 1 - \int_0^1 e^{-\lambda p V_h t} dt - \int_0^1 \lambda p V_h e^{-\lambda p V_h t} dt = 1 - 2 \left( 1 - e^{-\lambda p V_h} \right)
\]

Eq. (20) follows the conditional probability theorem. Eq. (22) is based on the hard core process [16, Ch. 3.5].

Fig. 8. (a) Gain of full-duplex over half-duplex ALOHA networks vs. link distance \(d\) for different \(n\). Note \(n\) is related to node density as \(n/\pi R^2 = \lambda p\). (b) Gain of full-duplex over half-duplex ALOHA networks vs. \(n\) for different link distance \(d\).

Fig. 9. (a). Gain under different carrier sensing schemes of full-duplex network (b). Density of Successful Transmissions under \(d \sim \text{unif}[0, 80]\) for Half-duplex networks with perfect carrier sensing and Full-duplex CSMA networks.

In Fig. 8, we show the full-duplex gain in ALOHA networks under the same simulation settings. It is clear that when link distance \(d\) increases to \(R_T\), the gain drops quickly toward 0. In other words, full-duplex ALOHA networks have even much lower mean throughput than the half-duplex counterparts, when mean link distance and node density go beyond a certain threshold. The fundamental reason is that full-duplex networks have much heavier interference footprints than half-duplex networks, since both primary and secondary transmitters are occupying their own interference region and suppressing others. This becomes more significant when either link distance \(b\) or density \(n\) increases.

We plot the analytical full-duplex gain for all cases for CSMA-based network in Fig. 9, using similar parameters as above (Note that \(R_T = R_C = 100m\)). A common observation is that the gain may be close to 2 when \(d\) is near 0, but decreases as \(d\) approaches \(R_T\). The reason behind is the same as the decreasing transmission density as discussed above. Fig. 7 also shows that the deployment density \(\lambda p\) has minor impact under perfect carrier sensing: for a given \(d\), the gain quickly saturates as \(\lambda p\) increases. We found the imperfect sensing and RTS/CTS cases show similar behavior.

Among all cases (Fig. 9(a)), the gain drops fastest in the perfect carrier sensing case. The underlying reason can be understood from Fig. 3(a). For a half-duplex network with perfect carrier sensing, as \(d\) approaches \(R_T\), a larger fraction of space can be reused by neighboring links, which diminishes the full-duplex’s advantage in concurrent transmissions. The trend is consistent with the protocol model in [10]. On the other hand, the imperfect carrier sensing and RTS/CTS cannot fully take advantage of the spatial reuse, thus amplifying the full-duplex gain at larger \(d\). We found that for the largest \(d\) \((d = R_T)\) in the former two cases, the full-duplex gain is 1.4 and 1.71, and for the largest \(d\) \((d = R_S)\) in the RTS/CTS case it is 1.88.
Our numerical calculation reveals that $P(\mathcal{H}|p)$ increases slightly with node density, but is less than 4.7% even in highly dense networks with $n = 20$ per $100^2 \text{ m}^2$, and even when $d$ is set to the maximum range of 100m. This implies that the two link case suffices to represent the impact of hidden terminals. The intuition behind such low probability $P(\mathcal{H}|p)$ is that, if more than two potential transmitters fall within the small $V_h$ region, very likely they can sense each other and at most one will become inactive after carrier sensing. For RTS/CTS, the same analysis and result holds, which we omit for brevity. Since $V_h$ for RTS/CTS is less than that of imperfect carrier sensing, the corresponding probability for RTS/CTS is also less than that of imperfect sensing.

2) Optimal Medium Access Probability: In the ALOHA network, each transmitter access the medium with probability $p_m$, and it is interesting to see what is the optimal $p_m$ value for both half-duplex and full-duplex network. In Fig. 10, we plot the successful density under different medium access probabilities $p_m$. There are two observations: (1), when $p_m$ increases, the successful density first increase as well, after reaching the optimal points, it start to decrease. This can be understand, since, when $p_m$ is relative small, it will cause less interference. However, when $p_m$ is large enough, it cause huge interference due to uncoordinated transmission. (2), the optimal successful density for half-duplex and full-duplex is achieved with different $p_m$ value. Next, we will explore the optimal $p_m$ from theoretical aspects.

Consider the successful density for half duplex is thus given by: $\lambda_{HA} = p_m \lambda_p \exp(-p_m \lambda_p V_{HP})$. To obtain the optimal $p_m$ value, we need to calculate $\frac{\partial \lambda_{HA}}{\partial p_m} = 0$. By solving this equation, $p_m^{opt} = 1/(\lambda_p V_{HP})$. Under this optimal $p_m^{opt}$, the optimal successful density is $\lambda_{HA}^{opt} = 1/(eV_{HP})$. Similarly, for full duplex ALOHA network, the optimal medium access probability is $p_F^{opt} = 1/(\lambda_p V_F)$, and the optimal successful density for full-duplex is $\lambda_F^{opt} = 1/(eV_F)$.

From the theoretical derivation, we find optimal $p_m$ depends on original point density $\lambda_p$ as well as contention region. Also, if every transmitter use optimal $p_m$ value, then the optimal successful density $\lambda_{opt}$ only depends on contention region. Furthermore, the optimal gain of full-duplex over half-duplex equals $\lambda_{opt} = 2\lambda_{HA}^{opt}/\lambda_F^{opt} = 2V_F/V_{HP}$. We plot the optimal gain under different link distance in Fig. 11, and when $d$ increases, the optimal gain actually decreases.

3) Beyond Fixed Link Distance: All the analysis above has assumed the same fixed distance $d$ for all links. We now show that the spatial throughput is the same as that under a random uniformly distributed link distance with mean $d$.

Denote the link distance of the typical link as $r_1$ and a contending link as $r_2$. Unlike the previous MCR analysis, we should calculate the MCR conditioned on $r_1$ and $r_2$. Denote the corresponding MCR as $V(r_1, r_2)$. Suppose link distance is uniformly distributed from $[0, R_S]$, where $R_S$ is the transmission range, we can obtain the density of successful transmission by averaging over all possible $r_1$ and $r_2$ as,

$$\frac{1}{R_S} \int_0^{R_S} \int_0^{R_S} \frac{1 - e^{-\lambda_p V(r_1, r_2)}}{V(r_1, r_2)} dr_1 dr_2 \quad (24)$$

This applies to both the full-duplex and half-duplex cases. We now numerically compare the resulting transmission density with simulation, following similar setup as in the fixed-$d$ case (Sec. III-B.3). Fig. 9(b) shows that the results from Eq. (24) are in close agreement with simulation. We only show perfect carrier sensing for half-duplex, without loss of generality. Given $R_S = 80m$ (mean link distance $d = 40m$), we also our previous fixed distance model with that of the uniformly distributed case in Fig. 9(b) and we can see that the results match closely.

4) Full-Duplex Cut-Through Transmission Mode: Using some elements of the foregoing analysis, and by extending the Poisson Bipolar model, we now prove that the cut-through transmission is always inferior to the bi-directional transmission mode. Denote the transmitter $T$, primary receiver $R_1$ and secondary receiver $R_2$ (Fig. 1) using their locations $X_i$, $Y_i$ and $Z_i$, respectively. We assume the $X_i$ follows a PPP; $Y_i$ is at distance $d$ with a random orientation $\theta$, uniformly distributed in $[0, 2\pi]$. $\gamma_i$ denotes the orientation of the secondary receiver w.r.t. the primary receiver. Since we need to ensure that $Z_i$ is not interfered by $X_i$, the range of link distance $d$ should be $[\frac{\theta}{2\cos(\theta)}, R_1]$ and the range of $\gamma_i$ should be $[\theta - \arccos(\frac{R_1^2}{d^2}) - 1, \theta + \arccos(\frac{R_1^2}{d^2}) - 1]$ in which it is uniformly distributed. Similar to the bi-directional full-duplex model we assume $X_i$ and $Y_i$ have the same back-off counter, which are uniformly distributed in $[0, 1]$. With this model setup, we can prove that:

**Proposition 1:** The network throughput under bi-directional transmission mode is no smaller than that of cut-through transmission mode with either perfect/imperfect carrier sensing.

**Proof:** From Fig. 3(d), the condition for successful bi-directional full-duplex transmission is that the shaded region cannot be reused by other $T_1/2_2$, and we can obtain MCR for it as $V_F$ (Theorem 4). Based on Fig. 12 we elucidate the spatial occupation of cut-through transmission. Observing both the transmitter and secondary receiver operate in half-duplex mode. So we consider two situations here.
transmission probability equals the probability that its signal-to-interference ratio (SIR) exceeds a threshold $\beta$, conditioned on this link wins the CSMA or ALOHA contention.

1) Modeling Contention and SIR: For ALOHA contention, the winning transmitters can be modeled using Matérn Type I point process. Assuming that every transmitter accesses the channel with an independent medium access probability $p_m$, and $\lambda_s$ is the intensity of original PPP $\Phi_o$, then the winning transmitters in ALOHA networks should be a PPP $\Phi^H_m$ with density $p_m \lambda_p$.

We also leverage the Matérn Type II point process [18] to model the winning transmitters in half-duplex CSMA contention. A transmitter of the original deployed PPP $\Phi_o$ is chosen for the Matérn process if it has the least backoff counter among all other points that lie in its carrier sensing range $R_C$. Denote $\Phi^H_m$ as the thinned point process after all winning transmitters are chosen, then its intensity is [19]:

$$\lambda^b_m = 1 - e^{-\lambda_p \pi R_C^2}$$

where $\lambda_p$ is the intensity of $\Phi_o$, i.e., the deployment density.

In both situations, for a winning transmitter $X_i$, the receiver $Y$ can successfully decode its packets only if its SIR satisfies:

$$SIR_{Y_i} := \frac{P h_{X_i, Y} d^{-\alpha}}{\sum_{j \in \Phi^H_m, j \neq i} P h_{X_i, Y_j} \|X_j - Y_i\|^{-\alpha}} > \beta$$

where $P$ represents the transmission power and $h_{XY}$ represents the channel fading coefficient from a transmitter $X$ to a receiver $Y$. We assume the Rayleigh fading model, in which the $\{h_{XY}\}$ are a set of i.i.d. exponentially distributed random variables with mean $E(h_{XY}) = 1$. The path loss from a node $x$ to a node $y$ is given by $\|x - y\|^{-\alpha}$, where $\|x - y\|$ is the Euclidean distance between the nodes and $\alpha$ is the path loss exponent.

2) Successful Transmission Probability: For ALOHA contention, the intensity of successful transmitters has been already established in previous work [20]. For completeness, we restate the result [20]:

**Theorem 6:** Under the physical model with ALOHA contention, the successful transmission probability is given by:

$$P_{\Phi^H_m} (SIR_{Y_i} > \beta) = \exp \left( -\lambda_p p_m d^\alpha \beta^2 C(\alpha) \right)$$

where $C(\alpha) = \frac{2 \pi^2}{\alpha \sin(\pi \alpha)}$.

For CSMA contention model, we consider a typical transmitter $X_o \in \Phi^H_m$, located at the origin $o$, and receiver $Y_o$ at a distance $d$ with orientation $\phi$ uniformly distributed in $[0, 2\pi)$. Then we calculate the probability in Eq. (26) under the Reduced Palm distribution $P_{\Phi^H_m}^{o_{\phi}}$ of $\Phi^H_m$, since Reduced Palm distribution is used when we do not want to consider a point that a point process conditioned on (16, Ch. 8.4). $P_{\Phi^H_m}^{o_{\phi}} (SIR_{Y_o} > \beta)$ denotes the probability that the SIR at receiver is greater than $\beta$ given that $X_o \in \Phi^H_m$, but not counting $X_o$’s transmission as interference. With this setup, we can have:

**Theorem 7:** Under the physical model with CSMA contention, the successful transmission probability for a typical
Poisson bipolar distributed half-duplex link can be bounded as:

\[
\Pr_m^{(o)}(SIR_{Y_o} > \beta) > \frac{1}{2\pi} \int_{0}^{2\pi} \exp \left\{ -\frac{1}{\ln 2} \int_{0}^{\infty} \int_{0}^{2\pi} k(r, \theta) \Delta(r, \theta, \phi) r \, dn \right\} d\phi,
\]

where \( k(r, \theta) \) denotes the probability that two transmitters of \( \Phi_o \) separated by a distance \( r \) and having a phase angle difference \( \theta \) are retained in \( \Phi_m^o \) and is given in [19]. \( V(r) \) is the union of areas covered by the carrier sensing ranges of the two transmitters separated by a distance \( r \) and is given by \( V(r) = 2\pi R_C^2 - 2R_C^2 \arccos(\frac{\tan \phi}{\ln 2}) + r \sqrt{R_C^2 - \frac{d^2}{2}} \), \( 0 \leq r \leq 2R_C \) and, \( \Delta(r, \theta, \phi) = \ln \left( 1 + \beta \left( \frac{d^2}{r^2 + d^2 - 2dv \cos(\theta - \phi)} \right)^{\frac{\alpha}{2}} \right) \).

**Proof:** We first calculate the probability of SIR coverage at the receiver conditioned on its orientation \( \phi \) with the horizontal, i.e., we calculate \( \Pr_m^{(o)}(SIR_{Y_o} > \beta | \phi) \). Then, in the end we get \( \Pr_m^{(o)}(SIR_{Y_o} > \beta) \) by deconditioning on \( \phi \). To reduce notational complexity, in this proof we will henceforth refer to probability of coverage conditioned on \( \phi \) as \( \Pr_m^{(o, \phi)}(SIR_{Y_o} > \beta) \).

\[
\Pr_m^{(o, \phi)}(SIR_{Y_o} > \beta) = \Pr_m^{(o, \phi)} \left( \sum_{X_i \in \Phi_m^o} Ph_{X_i,Y_o} \|X_i - Y_o\|^{-\alpha} > \beta \right)
\]

\[
= \Pr_m^{(o, \phi)} \left( \prod_{X_i \in \Phi_m^o} \mathbb{E}_{h_{X_i,Y_o}} \left[ \exp \left\{ -\beta d^\alpha \sum_{X_j \in \Phi_m^o} h_{X_j,Y_o} \|X_j - Y_o\|^{-\alpha} \right\} \right] \right)
\]

\[
= \Pr_m^{(o, \phi)} \left( \prod_{X_i \in \Phi_m^o} \mathbb{E}_{h_{X_i,Y_o}} \left[ \exp \left\{ -\beta d^\alpha \sum_{X_j \in \Phi_m^o} h_{X_j,Y_o} \|X_j - Y_o\|^{-\alpha} \right\} \right] \right)
\]

\[
> \exp \left\{ -\Pr_m^{(o, \phi)} \left( \prod_{X_i \in \Phi_m^o} \mathbb{E}_{h_{X_i,Y_o}} \left[ \exp \left\{ -\beta d^\alpha \sum_{X_j \in \Phi_m^o} h_{X_j,Y_o} \|X_j - Y_o\|^{-\alpha} \right\} \right] \right) \right\}
\]

\[
= \exp \left\{ -\Pr_m^{(o, \phi)} \left( \sum_{X_i \in \Phi_m^o} \Delta(r, \theta, \phi) \right) \right\}
\]

\[
= \exp \left\{ -\frac{1}{\ln 2} \int_{0}^{\infty} \int_{0}^{2\pi} \rho^{(2)}(r, \theta, \phi) r \, dn \right\}
\]

\[
= \exp \left\{ -\frac{1}{\ln 2} \int_{0}^{\infty} \int_{0}^{2\pi} k(r, \theta) \Delta(r, \theta, \phi) r \, dn \right\}
\]

To obtain Eq. (28), we condition on the point process \( \Phi_m^o \) and \( h_{X_i,Y_o} \) and use the fact that \( h_{X_i,Y_o} \) is an exponential random variable with \( \mathbb{E}(h_{X_i,Y_o}) = 1 \). As \( \{h_{X_i,Y_o}\} \) is a set of i.i.d random variables, we get Eq. (29). To get Eq. (30),

\[
SIR_{X_i} := \frac{Ph_{Y_i,X_i} d^{-\alpha}}{\sum_{(X_j,Y_j) \in \Phi_m^o,j \neq i} Ph_{X_j,X_i} \|X_j - X_i\|^{-\alpha} + Ph_{Y_j,X_i} \|Y_j - X_i\|^{-\alpha}} > \beta
\]

\[
SIR_{Y_j} := \frac{h_j d^{-\alpha}}{\sum_{q_j \in \Phi_m^o,j \neq j} h_j \left( \|X_j - Y_j\|^{-\alpha} + \|Y_j - Y_i\|^{-\alpha} \right)} > \beta
\]
As the process $\Phi_m^F$ is a Poisson point process, according to Slinyak’s Theorem [22], we can add a reference primary transmitter, say $X_o$, to the process with its secondary transmitter, say $Y_o$, at the origin. The probability that two nodes in a typical full-duplex pair $(X_o, Y_o)$ successfully transmit to each other is then given by $\mathbb{P}_{\Phi_m^F} (SIR_{X_o} > \beta, SIR_{Y_o} > \beta)$, which we derive as follows.

1) ALOHA Networks: In the case of full-duplex networks under ALOHA contention, we consider that every pair contends for channel access with a medium access probability $p_m$ independent of the other pairs. That is, if a pair wins channel contention, both the nodes in the pair transmit synchronously to each other. Similarly, the winning transmitters should be a PPP $\Phi_m^F$ with density $p_m \lambda_p$, if the primary transmitters are distributed as homogeneous PPP with density $\lambda_p$.

**Theorem 8:** Under the physical model with ALOHA contention, the successful transmission probability for a typical full-duplex bi-directional link can be bounded by:

$$
\mathbb{P}_{\Phi_m^F} (SIR_{X_o} > \beta, SIR_{Y_o} > \beta) 
\leq \exp \left\{ -2p_m \lambda_p \int_0^\infty \frac{r}{1 + (\beta d^\alpha - 1) (r - \alpha + \beta d^\alpha r - \alpha + d - \alpha)^{-1}} dr \right\}
$$

**Proof:** The probability that two nodes in a pair successfully transmit to each other can be upper bounded as,

$$
\mathbb{P}_{\Phi_m^F} (SIR_{X_o} > \beta, SIR_{Y_o} > \beta) \leq \mathbb{P}_{\Phi_m^F} (SIR_{Y_o} > \beta)
$$

(33)

We note here that we could have upper bounded the probability that two nodes in a pair successfully transmit to each other by $\mathbb{P}_{\Phi_m^F} (SIR_{X_o} > \beta)$ equivalently. As we consider a symmetric channel between $X_o$ and $Y_o$, and the quantities $\mathbb{P}_{\Phi_m^F} (SIR_{X_o} > \beta)$ and $\mathbb{P}_{\Phi_m^F} (SIR_{Y_o} > \beta)$ are spatial averages this would not make a difference. Making similar assumptions about the channel fading coefficients as in the analysis of full-duplex CSMA networks, we have,

$$
\mathbb{P}_{\Phi_m^F} (SIR_{Y_o} > \beta) = \mathbb{E}_{\Phi_m^F} \left[ \sum_{q_j \in \Phi_m^F} h_j^o \left( \|X_j - Y_o\|^{-\alpha} + \|Y_j - Y_o\|^{-\alpha} \right)^{-1} > \beta \right]
$$

$$
= \mathbb{E}_{\Phi_m^F} \left[ h_o > \beta d^\alpha \sum_{q_j \in \Phi_m^F} h_j^o \left( \|X_j\|^{-\alpha} + \|Y_j\|^{-\alpha} \right) \right]
$$

$$
= \mathbb{E}_{\Phi_m^F, h_j^o} \left[ \exp \left\{ -\beta d^\alpha \sum_{q_j \in \Phi_m^F} h_j^o \left( \|X_j\|^{-\alpha} + \|Y_j\|^{-\alpha} \right) \right\} \right]
$$

$$
\leq \mathbb{E}_{\Phi_m^F, h_j^o} \left[ \exp \left\{ -\beta d^\alpha \sum_{X_j \in \Phi_m^F} h_j^o \left( \|X_j\|^{-\alpha} + \|X_j\| + d\right)^{-\alpha} \right\} \right]
$$

(34)

$$
= \mathbb{E}_{\Phi_m^F} \left[ \mathbb{E}_{h_j^o} \left[ \prod_{X_j \in \Phi_m^F} \exp \left\{ -\beta d^\alpha h_j^o \left( \|X_j\|^{-\alpha} + \|X_j\| + d\right)^{-\alpha} \right\} \right] \right]
$$

(36)

$$
= \mathbb{E}_{\Phi_m^F} \left[ \prod_{X_j \in \Phi_m^F} \mathbb{E}_{h_j^o} \left[ \exp \left\{ -\beta d^\alpha h_j^o \left( \|X_j\|^{-\alpha} + \|X_j\| + d\right)^{-\alpha} \right\} \right] \right]
$$

(37)

We get Eq. (34) by replacing $Y_o$ with 0 as $Y_o$ is located at the origin. To get Eq. (35) we use the fact that $h_o$ is an exponentially distributed random variable with $\mathbb{E}(h_o) = 1$. By replacing $\|Y_j\|^{-\alpha}$ in Eq. (36) with $\left( \|X_j\| + d\right)^{-\alpha}$ in Eq. (37), we get a lower bound on the total interference at the origin (which is the sum in Eq. (36)), which gives the upper bound in Eq. (37). We obtain Eq. (38) using the fact the $\{h_j^o\}$ is a set of i.i.d random variables. We obtain Eq. (39) using the probability generating functional of the Poisson point process $\Phi_m^F$ [23]. Finally, by combining Eq. (39) with (33) we get the statement of Theorem 8.

2) CSMA-Based Networks: Similarly, we derive the successful transmission probability for full duplex bi-directional link under CSMA contention as follows.

**Theorem 9:** Under the physical model with CSMA contention, the successful transmission probability for a typical full-duplex bi-directional link can be approximated by:

$$
\mathbb{P}_{\Phi_m^F} (SIR_{X_o} > \beta, SIR_{Y_o} > \beta) 
\approx \frac{1}{2\pi} \exp \left\{ -\frac{L^2}{2\pi \lambda_m^F} \int_0^{2\pi} \int_0^{2\pi} k(r, \theta, \phi, \delta) \Delta(r, \theta, \phi, \delta) r dr d\theta d\delta \right\} d\phi
$$

where, $k(r, \theta, \phi, \delta)$ denotes the probability that two pairs $q_i$ and $q_j$ of $\Phi_o$ are retained in $\Phi_m^F$ and is given by,

$$
k(r, \theta, \phi, \delta) = \begin{cases} 
0, & (r, \theta, \phi, \delta) \in B_1 \cup B_2 \cup B_3 \cup B_4 \\
2g(V_1), & \text{otherwise}
\end{cases}
$$

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.
where $\lambda$ is the density of node locations, $\beta$, $SIR$, $Y$, and $V$ are parameters defined in the problem formulation. For the half-duplex CSMA networks, denoted as $C$, the throughput can be calculated as the product of the intensity of concurrent transmission pairs and every pair has two active transmissions.

Proof: Available in the technical report VI-C.

In Fig. 13 (b) we compare the simulation results for the density of successful transmissions of full-duplex CSMA networks and the above approximation. We simulate 20 topologies and observe that the estimation obtained is in close agreement with the simulation results across different $d$ and $\lambda_p$ settings.

C. Full-Duplex Gain Under Physical Model

Under the physical model, the spatial network throughput can be calculated as the product of the intensity of concurrent transmissions and the probability that a transmission is successful. Here, we only focus on networks gain under CSMA contentions, since the full-duplex gain under ALOHA contention can be derived in a similar way. The throughput for the half-duplex CSMA networks, denoted as $T_H$, is thus given by,

$$T_H = \lambda_h^m \Phi_m^\alpha (SIR_{Y_o} > \beta)$$

where $\lambda_h^m$ is given by Eq. (25) and $\Phi_m^\alpha (SIR_{Y_o} > \beta)$ can be obtained from the result in Theorem 7.

For the full-duplex CSMA networks, the network throughput can be derived as,

$$T_F = 2\lambda_f^m \Phi_f^\alpha (SIR_{X_o} > \beta, SIR_{Y_o} > \beta)$$

where $\lambda_f^m$ is given by Eq. (18) and $\Phi_f^\alpha (SIR_{X_o} > \beta, SIR_{Y_o} > \beta)$ can be obtained following Theorem 9. There is multiplication by a factor 2 because $\lambda_f^m$ is the density of transmission pairs and every pair has two active transmissions.

In Fig. 14 we plot the network throughput gain of full-duplex over half-duplex CSMA networks, $T_F/T_H$, with varying link distance $d$ and deployment density $\lambda_p$ where $\lambda_p = n/\pi R_S^2$. We observe that the full-duplex gain shows similar trend as in the protocol model, i.e., it approaches 2 as $d$ increases and decreases to around 1.4 as $d$ approaches $R_S$.

We also observed that the carrier sensing range $R_C$ is a crucial parameter that affects the spatial throughput. Fig. 16(a) plots the numerical spatial throughput under different $R_C$ settings, with SIR threshold $\beta = 10$ dB, $\alpha = 4$ and maximum link distance 100m. A smaller $R_C$ may not be able to protect the receiver from interference, whereas a larger $R_C$ degrades spatial reuse between links. Remarkably, the throughput-optimal $R_C$ for full-duplex is smaller than that of half-duplex, again because of full-duplex’s capability of perfect carrier sensing. Each full-duplex receiver itself is a transmitter, and hence it does not require the other transmitter to extend $R_C$ to protect it. From Fig. 16 (b) we can see that under the optimal $R_C$ values the full-duplex can provide a throughput gain larger than 2 for smaller link distances (e.g., 2.3× for $d = 10m$). This is another aspect that network planners need to consider when designing full-duplex radios. Given the consistency of physical and protocol model, we expect a varying $R_C$ will affect the full-duplex gain in the protocol model as well. We leave the detailed exploration for future work.

V. RELATED WORK

Recent research on full-duplex mainly focuses on implementing new radio hardware architecture and signal processing algorithms. Choi et al. [2] is the first to realize single-channel full-duplex over ZigBee radios, which inspired substantial follow-on efforts. In particular, Bharadia et al. [3] recently implemented the first in-band full-duplex WiFi radio with a single antenna.

The development of full-duplex radios is marching steadily towards commercialization [1]. In contrast, the impact of full-duplex on higher network layers remains largely unexplored. Centralized scheduling and decentralized random access protocols [4], [5] have been proposed for full-duplex wireless LANs. Each of these protocols modifies the current CSMA MAC to capitalize on the concurrent transmission/reception capability. The performance limit of full-duplex and its dominating factors are yet to be investigated.

Since the landmark paper of Gupta and Kumar [6], substantial effort has focused on analyzing wireless network capacity under various topologies and PHY layer technologies [24]. Existing analysis mostly assumes half-duplex radios, and targets capacity scaling laws under an asymptotically growing node population. Information theoretic properties of single-cell full-duplex WLAN are analyzed recently [25]. Yet it remains an open problem what is the fundamental gain when full-duplex interplays with multi-cell interference and spatial reuse.

Recent work of Yang et al. [26] used a simplified unit-disk model to compare the asymptotic dominance relation (higher or lower) between MIMO and full-duplex radio modes. Xie and Zhang [10] are the first to derive an upperbound of full-duplex network capacity through disk-packing. In contrast, our approach leverages stochastic geometry for an average-
case analysis of the full-duplex capacity and its gain over half-duplex. Our analysis shows consistent trend with [10], i.e., the gain decreases as link distance approaches interference range. However, our framework can analyze the impact of a more comprehensive set of network parameters, carrier sensing models, and interference model. Our analysis also explains why the practical gain of full-duplex can exceed the upperbound in [10] that does not consider carrier sensing artifacts (caused by hidden/exposed terminals).

More recently, Tong and Haenggi [27] analyzed throughput of ALOHA for full-duplex and half-duplex hybrid network. Atzoni and Kountouris [28] analyze full-duplex MIMO, and also testifies full-duplex gain of less than 2 for realistic scenarios. Mohammadi et al. [29] analyze networks with full-duplex APs supporting simultaneous downlink and uplink, and show that even with imperfect self-interference cancellation, full-duplex network can still outperform half-duplex. Wang et al. [30] reveal severe aggregate interference affects full-duplex capacity gain, and prove that self-interference cancellation alone is not enough to scalable full duplex gain. In [31], Qin et al. consider end-to-end throughput in multi-hop wireless networks, and show that full-duplex can achieve throughput gain of more than 2 under oracle centralized scheduling. All above-mentioned work focus on ALOHA networks, but our work consider the more practical cases for CSMA networks, which is non-trivial especially consider the sophisticated models for imperfect carrier sensing and RTS/CTS.

Stochastic geometry has shown great potential in quantifying the spatial reuse in wireless networks [7]. Particularly to IEEE 802.11 networks, the key analytic question lies in approximating the sparsified winning node distribution after contention [15]. In [8], an HCPP model is proposed to capture key properties of 802.11 networks (for Poisson node distribution). Alfano et al. [32] extended model to analyze the nodes’ throughput variation under a minimum link-distance constraint. Substantial work has focused on more accurate approximation of contention behavior [15]. Our recent analytical framework in [33] represents the first to advance stochastic geometry to analyze full-duplex wireless networks. Our analysis overcomes the limitations of classical stochastic geometry (Sec. II), and enables an investigation of different carrier sensing schemes for both full- and half-duplex networks. In the present work, we further enrich [33] with detailed models/proofs for the full-duplex gain under protocol/physical models, and for the asymptotic comparison of the two full-duplex modes (bi-directional transmission vs. cut-through transmission). We also establish a set of new analysis for ALOHA networks, which consolidates the important tradeoff between doubling link capacity and worsening spatial (interference) footprint.

VI. CONCLUSION

We have devised a stochastic geometry framework to analyze the spatial footprint of full-duplex and half-duplex links under perfect/imperfect carrier sensing and RTS/CTS signaling. Our framework introduces a new analytic tool, i.e., the mean contention region, that integrates classical stochastic geometry with the protocol interference model. This allows us to establish closed-form formulas for the full-duplex gain under different topological properties and protocol imperfectness. Our analysis shows that, if carrier sensing is perfect, then the full-duplex gain approaches 2 only when inter-link interference is negligible (link distance approaches 0 and network density approaches 1). A broader implication is that the full-duplex gain diminishes a lot in centrally scheduled networks (no carrier sensing artifact) and multi-hop networks (link distance needs to be long to satisfy asymptotic connectivity constraint). In addition, if the hardware cost is the same, then MIMO is always better than full-duplex as it increases link range. However, our framework can analyze the impact of different topological properties and protocol imperfectness.
Proof: We consider three regions shown as different patterns in Fig. 4(c) around typical link $L_o$.

First, the union of primary transmitter $T_1$’s and secondary transmitter $T_2$’s interference range ($V_1$), should be counted as a deterministic contention region.

Second, consider a contending link $L'$ whose primary transmitter $T_1'$ exists within the grey area in Fig. 16(a), then its associated secondary transmitter $T_2'$ may be located within $T_2$’s interference range, which leads to contention with $L_o$. The derivation of this spatial averaged contention region ($V_2$) is similar to Theorem 1. The symmetric region on the RHS in the same figure can be calculated in the same way.

There is a small area (shaded area in Fig. 16(b)) that needs special consideration: if another primary transmitter $T_3'$ resides there, its secondary transmitter $T_2'$ could fall in $T_1$ or $T_2$’s interference range. For the LHS, we only need to consider $T_2'$ is located within $T_1$’s interference range; and similarly RHS for $T_2$’s interference range.

Third, consider a contending link $L'$ with its transmitter lying in the upper shaded area in Fig. 16(c). The associated secondary transmitter $T_2'$ can reside in both primary transmitter $T_1$ and secondary transmitter $T_2$’s interference range. This is different from situation 2 as the potential location of $T_2'$ is continuous, which corresponds to $V_3$.

Finally, any area outside the above cases falls outside the potential contention region of the typical full-duplex link.

C. Proof of Theorem 9

Proof: We first introduce the following definitions, considering the two full-duplex pairs $q_i$ and $q_j$ shown in Fig. 17.

(a) $B_1$ to be the set of $(r, \theta, \phi, \delta)$ values such that $X_j$ and $X_i$ are within $R_C$ of each other. We similarly define $B_2, B_3, B_4$ for the cases when $X_j$ and $Y_i$ are within $R_C$ of each other, and $Y_j$ and $X_i$ are within $R_C$ of each other, respectively. Thus we have,

\[ B_1 = \{ (r, \theta, \phi, \delta) : r \leq R_C \} \]
\[ B_2 = \{ (r, \theta, \phi, \delta) : r^2 + d^2 - 2rd \cos(\theta - \phi) \leq R_C^2 \} \]
\[ B_3 = \{ (r, \theta, \phi, \delta) : r^2 + d^2 + 2rd \cos(\theta - \phi) \leq R_C^2 \} \]
\[ B_4 = \{ (r, \theta, \phi, \delta) : (r \cos \theta + d \cos \phi - d \cos \phi)^2 + (r \sin \theta + d \sin \phi - d \sin \phi)^2 \leq R_C^2 \} \]

(b) $V_1 (r, \theta, \phi, \delta)$ is the mean contention region around two pairs $q_i$ and $q_j$ that are contending with other full-duplex links.

\[
\Delta(r, \theta, \phi, \delta) = \ln \left( 1 + \frac{\beta}{2} d^\alpha \sum_{q_1 \in \Phi_m^o} h_{q_1}^\alpha \left\{ \| X_j - X_o \|^{-\alpha} + \| Y_j - X_o \|^{-\alpha} \right. \right. \\
\left. \left. + \| X_j - Y_o \|^{-\alpha} + \| Y_j - Y_o \|^{-\alpha} \right\} \right)
\]

Later in the proof, we get a lower bound on (40) using Jensen’s inequality which serves as an estimate of $P_{\Phi_m^o}(SIR_{X_o} > \beta, SIR_{Y_o} > \beta)$. We validate this estimate by comparing it to the simulation results. We first obtain (40) by conditioning it the orientation $\phi$ of the secondary transmitter of the typical pair. To reduce notational complexity, we will denote this conditioning in the Reduced Palm measure as $P_{\Phi_m^o_{\phi}}$. Then, in the end we obtain the estimate by deconditioning w.r.t $\phi$. Also, in the following lines of the proof we will denote the term $\{ \| X_j - X_o \|^{-\alpha} + \| Y_j - X_o \|^{-\alpha} + \| X_j - Y_o \|^{-\alpha} + \| Y_j - Y_o \|^{-\alpha} \}$ in (40) by $I_{\phi}$. Thus we have,

\[
P_{\Phi_m^o_{\phi}} \left( h_o > \frac{\beta}{2} d^\alpha \sum_{q_j \in \Phi_m^o} h_{q_j}^\alpha \right)
\]
\[
= P_{\Phi_m^o_{\phi}} \left[ \exp \left\{ - \frac{\beta}{2} d^\alpha \sum_{q_j \in \Phi_m^o} h_{q_j}^\alpha \right\} \right]
\]
\[
= \prod_{q_j \in \Phi_m^o} \mathbb{E} h_{q_j}^\alpha \left[ \exp \left\{ - \frac{\beta}{2} d^\alpha \sum_{q_j \in \Phi_m^o} h_{q_j}^\alpha \right\} \right]
\]
\[
= P_{\Phi_m^o_{\phi}} \left[ \exp \left\{ - \sum_{q_j \in \Phi_m^o} \ln \left( 1 + \frac{\beta}{2} d^\alpha \right) \right\} \right]
\]
\[
\left\{\begin{array}{l}
> \exp \left\{ -\frac{1}{\Phi_m} \sum_{q_i \in \Phi_m} \ln \left( 1 + \frac{\beta}{2} d^2 j^2 \right) \right\} \\
= \exp \left\{ -\frac{1}{\Phi_m} \sum_{q_i \in \Phi_m} \ln \left( 1 + \frac{\beta}{2} d^2 \right) \left( \|X_j - X_o\|^{-\alpha} + \|Y_j - X_o\|^{-\alpha} + \|Y_j - Y_o\|^{-\alpha} \right) \right\} \\
\end{array}\right.
\]

(43)

(44)

To derive Eq. (41), we condition on the point process \( \Phi_m \) and \( h_o \) and use the fact that \( h_o \) is an exponential random variable with \( \mathbb{E}[h_o] = 1 \). As \( \{ h_j \} \) are a set of i.i.d random variables, Eq. (42) follows straightforwardly. To get Eq. (43), we apply Jensen's inequality using the fact that \( \exp(-.) \) is a strictly convex function. We get Eq. (44) by simply replacing \( l_j^2 \) with the terms in Eq. (40) mentioned previously. Eq. (45) is obtained by applying Campbell's theorem [17] to the previous step. To get Eq. (46), we use the relation between second-order product density of the stationary point process \( \Phi_m \) and \( k(r, \theta, \phi, \delta) \) based on [21]. Finally, by deconditioning Eq. (46) w.r.t \( \phi \), we get a lower bound on (40) which is the estimation in Theorem 9. We note that the function \( g(V_j) \) can be obtained in an analogous manner to the derivation of the function \( \eta(V_j) \) given the analysis of mean interference for Matérn type II process in Zhong et al. [9].

REFERENCES


Vignesh Venkateswaran received the B.Tech. degree in electrical engineering from the Indian Institute of Technology (IIT) Mandi, Suran, India, and the M.S. degree in electrical engineering from the University of Wisconsin-Madison, Madison, WI, USA. He is currently a Software Engineer with the Data Center Networking, Cisco Systems Inc., San Jose, CA, USA. His research interests include system software development for network analytics on next-generation data center networking switches. Xinyu Zhang received the Ph.D. degree in computer science and engineering from the University of Michigan in 2012. He is currently an Assistant Professor with the Department of Electrical and Computer Engineering, University of Wisconsin-Madison. His research interest lies in wireless sensing algorithms that facilitate the Internet-of-Things applications and radical architectures for next-generation wireless networks, based on millimeter-wave and large-scale MIMO. He received the ACM MobiCom Best Paper Award in 2011 and the NSF CAREER award in 2014.