Another view of Bayes

Light bulb factories

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>bulbs/day</th>
<th>% defective</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20000</td>
<td>30000</td>
<td>50000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>3%</td>
<td>1%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

sent A made by factory A

B defective

we have \( \Pr [B | A] = 5 \times 10^{-2} \) defective if made by A

made by A if defective

want \( \Pr [A | B] \)

\[
\Pr [A | B] = \frac{\Pr [B | A] \Pr [A]}{\Pr [B]}
\]

\[
\Pr [A] = \frac{20,000}{20,000 + 30,000 + 50,000} = 0.2
\]

\[
\Pr [B] = \sum \Pr [B | \text{machine}] = 5 \times 10^{-2} \left( \frac{20,000}{100,000} + 0.3 \times 10^{-2} \frac{30,000}{100,000} + 1 \times 10^{-2} \cdot \frac{50,000}{100,000} \right) = 0.24
\]

\[
\Pr [A | B] = \frac{5 \times 10^{-2} \times 0.2}{0.24} = \frac{5}{12}
\]

Had \( \Pr [A] \) → got more info
Asymptotics

**Big O**

\[ f, g : \mathbb{N} \to \mathbb{R} \]

\[ f(x) \text{ is } O(g(x)) \quad \text{[write } f(x) = O(g(x)) \text{] but NOT } ! \]

\[ \exists \ c, k, \forall x > k \quad |f(x)| \leq c |g(x)| \]

**Example**

\[ 3x^2 + x + 200 \text{ is } O(x^2) \]

\[ c = 204 \quad k = 1 \]

other choices...

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**Facts**

\[ |f(x)| = O(\log(x)) \implies f(x) = O(g(x)) \]

if \( f_1(x) \text{ is } O(g(x)) \)

\[ f_2(x) \text{ is } O(g(x)) \]

\[ f_1(x) + \alpha f_2(x) \text{ is } O(g(x)) \]

↑ Upper bound.

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**Def** \( f(x) \text{ is } \Omega(g(x)) \) if:

\[ \exists c, k \quad \forall x > k \quad |f(x)| \geq c |g(x)| \]

\[ c > 0 \]

**Def** \( f(x) \text{ is } \Theta(g(x)) \) if both \( O \) and \( \Omega \)

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**Guess & Verify**

Using calculus \( f(x) \text{ is } O(g(x)) \) if \( \limsup_{x \to \infty} \frac{f(x)}{g(x)} < \infty \)

[Prove!]
Limits of sequences
\{a_n \mid n \in \mathbb{N}\}

\[\text{Finite} \quad \lim_{n \to \infty} a_n = c \quad \text{if} \quad \forall \varepsilon > 0 \exists n_0 \in \mathbb{N} \quad \forall n \geq n_0 \quad |a_n - c| < \varepsilon\]

\[a_n \xrightarrow[n \to \infty]{} c \quad \text{if} \quad \lim_{n \to \infty} a_n = c\]

\[\text{Infinite} \quad \lim_{n \to \infty} a_n = \infty \quad \forall \varepsilon > 0 \exists n_0 \in \mathbb{N} \quad \forall n \geq n_0 \quad a_n \geq \varepsilon\]

- \[a_n, b_n \text{ real seq's} \quad a_n \sim b_n \quad a_n \text{ asymptotically equal to } b_n\]

\[\lim_{n \to \infty} \frac{a_n}{b_n} = 1 \quad (\text{if } \frac{a_n}{b_n} = \frac{0}{0} \text{ for a term use } l)

Equivalence Relation
\[a_n \sim b_n \rightarrow a_n = \Theta(b_n) \quad \text{but not conversely}.

If \(a_n, b_n > 0 \quad \forall n > n_0\)
\[a_n \sim b_n \quad \text{and} \quad c_n \sim d_n \Rightarrow a_n + b_n \sim c_n + d_n\]

but\[a_n = 1 + \frac{1}{n} \quad b_n = 1 \quad c_n = -1 \quad d_n = -1\]shows need for condition
$$a_n = o(b_n) \quad \text{little oh}$$

If

$$\lim_{n \to \infty} \frac{a_n}{b_n} = 0$$