State Complexity of Prefix Distance of Subregular Languages

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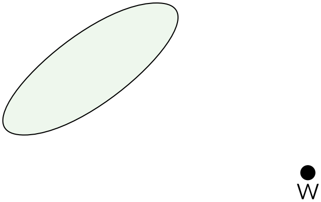
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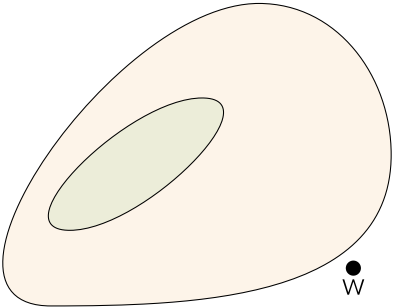
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- finite languages
- prefix-closed regular languages
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- finite languages
- prefix-closed regular languages
- prefix-free regular languages

The state complexity for these classes is strictly less than for regular languages.

A distance is a function $d : \Sigma^* \times \Sigma^* \to [0, \infty)$ such that

1. $d(x, y) = 0$ if and only if $x = y$
2. $d(x, y) = d(y, x)$
3. $d(x, y) \leq d(x, w) + d(w, y)$

The neighbourhood of a language $L \subseteq \Sigma^*$ of radius $k \geq 0$ with respect to a distance measure d is the set of all words u with $d(w, u) \leq k$ for some $w \in L$,

$$E(L, d, k) = \{u \in \Sigma^* : (\exists w \in L)\, d(w, u) \leq k\}.$$

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- The state complexity of these neighbourhoods is $(k+2)^n$
  - Upper bound (Salomaa, Schofield 2007)
  - Lower bound (Ng, Rappaport, Salomaa 2015)

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  - Lower bound (Ng, Rappaport, Salomaa 2015)
- Asymptotic lower bounds for neighbourhoods with respect to Hamming distance
  - $r = 1$ (Povarov 2007)
  - $r > 1$ (Shamkin 2011)

The prefix distance of x and y counts the number of symbols which do not belong to the longest common prefix of x and y.

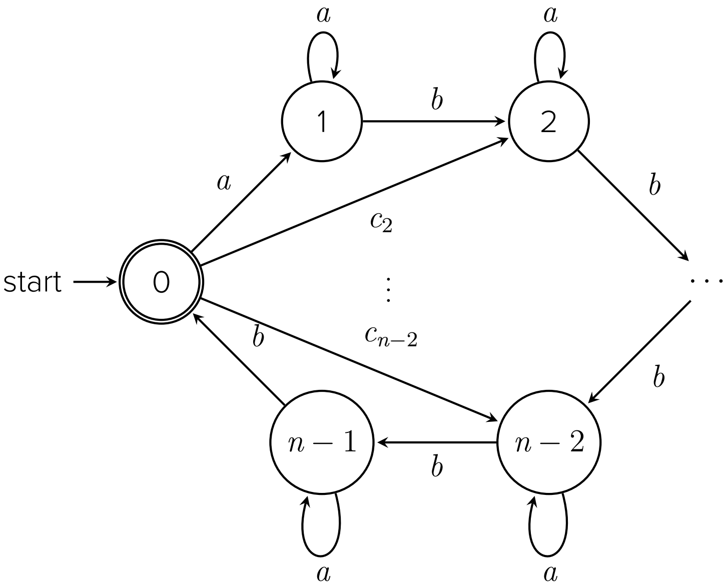
$$d_p(x, y) = |x| + |y| - 2 \cdot \max_{z \in \Sigma^*} \{|z| \mid x, y \in z\Sigma^*\}.$$

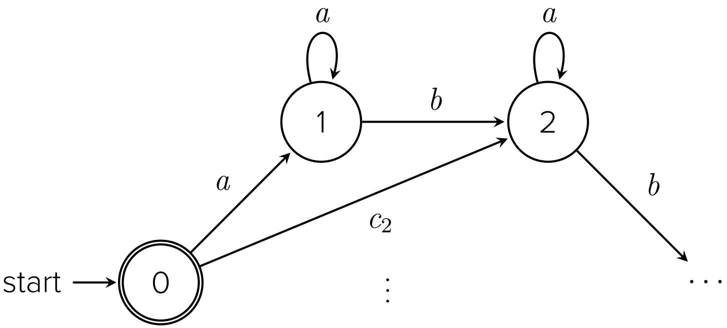
Harbord → Harbourfront

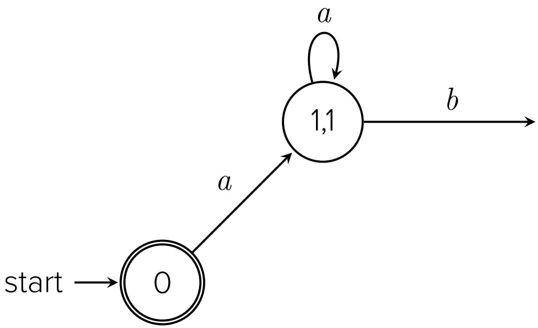
Theorem (Ng, Rappaport, Salomaa 2015)

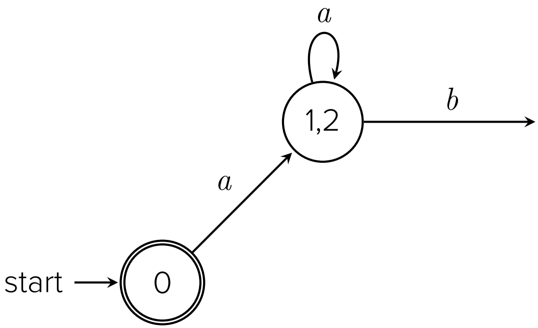
For $n > k \geq 0$, if $\mathrm{sc}(L) = n$ then

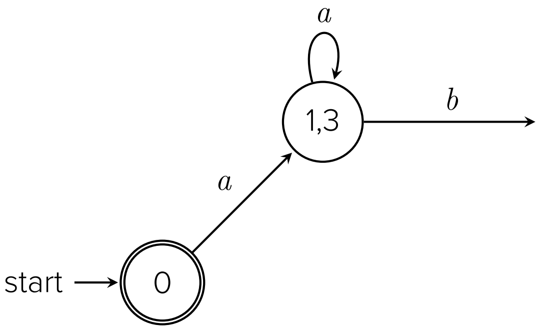
$$\mathrm{sc}(E(L, d_p, k)) \leq n \cdot (k+1) - \frac{k(k+1)}{2}.$$

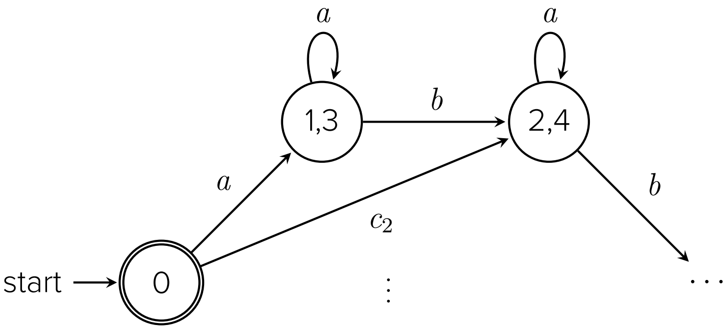

start
0
1
2
n − 1
n − 2
a
b
c2
cn−2
⋯
⋮

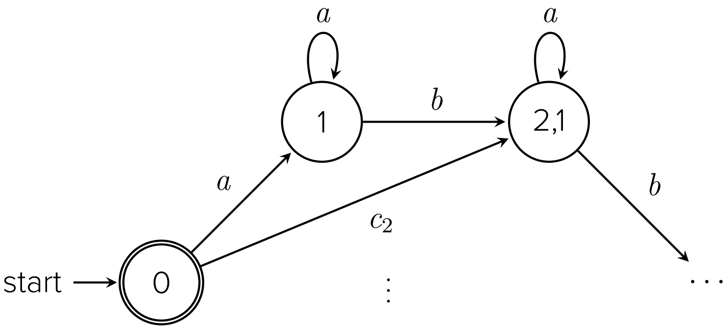

a
a
1
b
2
a
c2
b
start
0
⋮
…

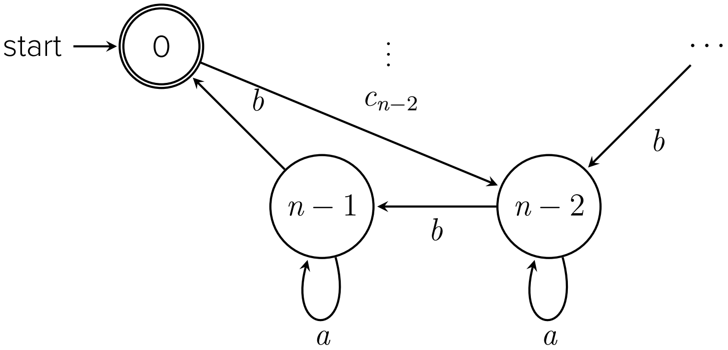

a
b
1,1
a
start
0

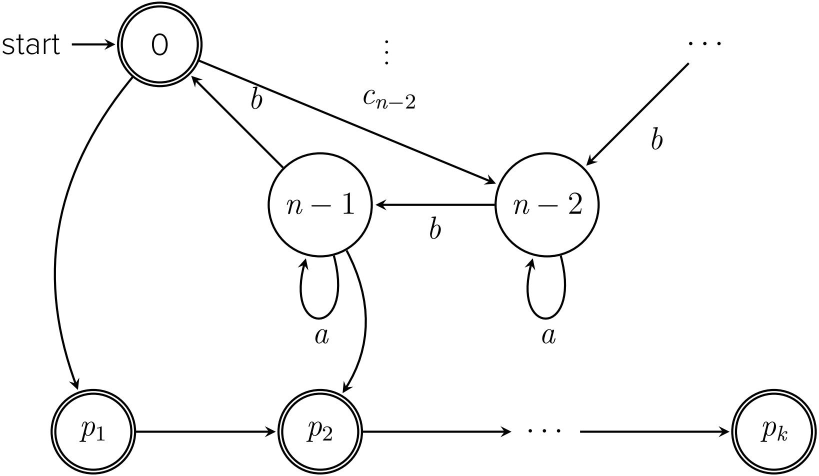

a
b
1,2
a
start
0


a
b
1,3
a
start
0


a
a
b
1,3
2,4
a
b
c_2
start
0
⋮
…


a
a
b
1
2,1
a
b
c2
start
0
⋮
⋯


start
0
$\vdots$
$\cdots$
b
c_{n-2}
b
$n-1$
$n-2$
b
a
a


start
0
⋮
c_{n-2}
⋯
b
b
$n-1$
$n-2$
b
a
a
p_1
p_2
⋯
p_k

This gives us $(n-f)\cdot(k+1)+k+f$ states in total, however not all of these states are reachable.

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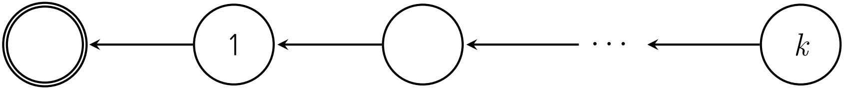
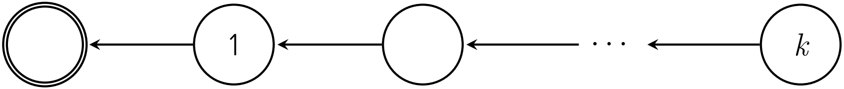


This gives us $(n - f) \cdot (k + 1) + k + f$ states in total, however not all of these states are reachable.



There are at least $1 + 2 + \cdots + k$ unreachable states. The number of reachable states is at most

$$n \cdot (k + 1) - \frac{k(k + 1)}{2}.$$

A language is finite if and only if it is recognized by an acyclic finite automaton.

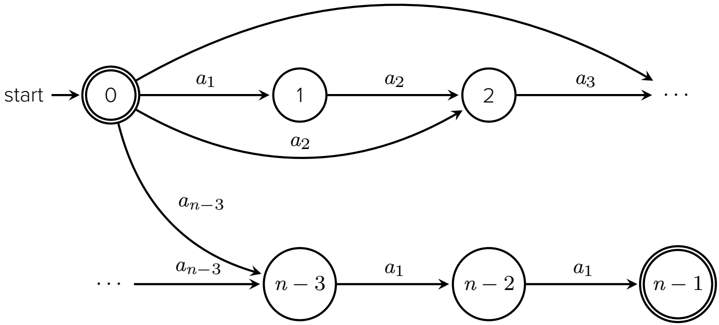
Theorem

Let L be a finite language. For $n > 2k \geq 0$, if $\mathrm{sc}(L) = n$, then

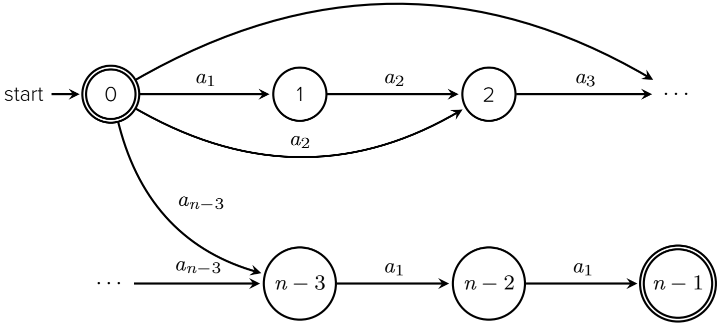
$$\mathrm{sc}(E(L, d_p, k)) \leq (n-2) \cdot (k+1) - k^2 + 2,$$

and this bound can be reached in the worst case.

Each state has a longest word that reaches it.



There must be at least 2 final states.



$$
\begin{aligned}
&(n-f)\cdot(k+1)+k+f-2\cdot\frac{k(k+1)}{2} \\
&=(n-2)\cdot(k+1)+k+2-k^2-k \\
&=(n-2)\cdot(k+1)+2-k^2
\end{aligned}
$$

A regular language is prefix-closed if $x \in L$ implies $p \in L$ for every prefix p of x.

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Theorem

Let L be a prefix-closed regular language recognized by an n-state DFA A. Then there is a DFA A' that recognizes the neighbourhood $E(L, d_p, k)$ with at most $n + k$ states and this bound is reachable.

Since every state is a final state, $f = n$ and

$$\begin{aligned}(n-f)\cdot(k+1)+f+k &= (n-n)\cdot(k+1)+n+k\\ &= n+k.\end{aligned}$$

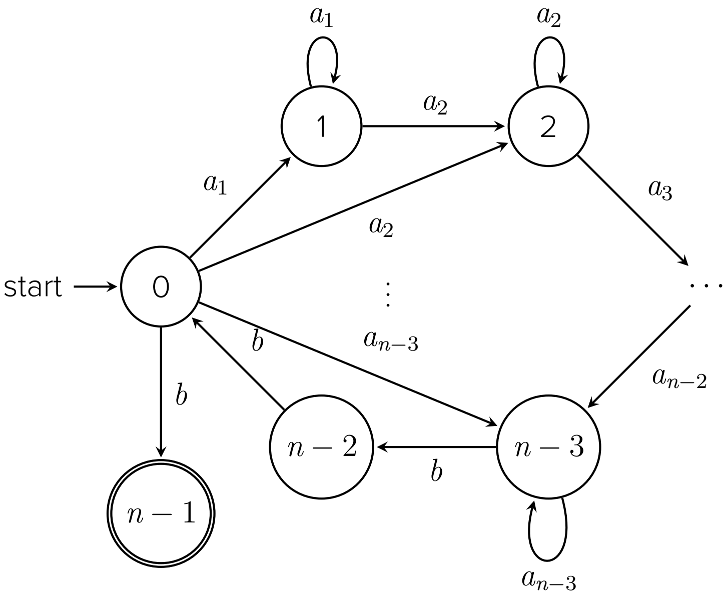
A language is prefix-free if for every $x \in L$, no prefix p of x is in L. A prefix-free regular language is recognized by a non-exiting finite automaton.

Theorem

Let L be a prefix-free regular language. For $n > k \geq 0$, if $\mathrm{sc}(L) = n$, then

$$\mathrm{sc}(E(L, d_p, k)) \leq (n-1) \cdot k + 2 - \frac{k(k-1)}{2},$$

and this bound can be reached in the worst case.



start
0
1
2
$\cdots$
$n-3$
$n-2$
$n-1$
a_1
a_2
a_3
$\vdots$
a_{n-3}
a_{n-2}
b

The state complexity neighbourhoods of some subregular language classes with respect to the prefix distance is strictly less than for regular languages.

Regular	$n \cdot (k+1) - \frac{k(k+1)}{2}$
Finite	$(n-2) \cdot (k+1) - k^2 + 2$
Prefix-closed	$n + k$
Prefix-free	$(n-1) \cdot k + 2 - \frac{k(k-1)}{2}$

Future work:

- Tight state complexity bounds for neighbourhoods of finite languages with respect to
  - additive distances
  - suffix and factor distances
- Other subregular language classes