Outfix-Guided Insertion

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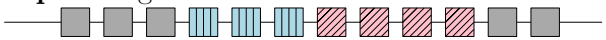
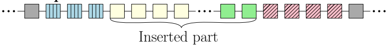
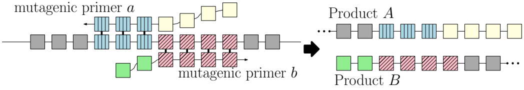
Input: A given DNA



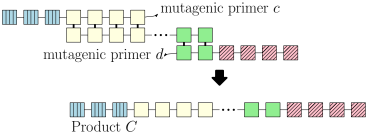
Output: A desired DNA



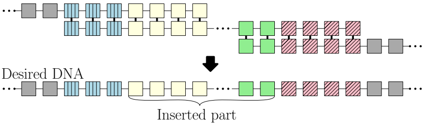
Step 1: Cut given DNA using primers a and b



Step 2: Annealing inserted sequence using primers c and d



Step 3: Ligation PCR with product A,B and C



Let $w, x, y, z \in \Sigma^*$. If $w = xyz$, we say x is a prefix of w, z is a suffix of w, and (x, z) is an outfix of w.

Classical insertion [Haussler 1983]

$$x \leftarrow y = \{x_1 y x_2 \mid x = x_1 x_2\}.$$

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Contextual insertion [Galiukschov 1981]

$$x \xleftarrow{C} y = \{x_1 u y v x_2 \mid (u, v) \in C, x = x_1 u v x_2\}$$

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Overlap assembly [Csuhaj-Varjú et al. 2007]

$$x \overline{\odot} y = \{uvw \in \Sigma^+ \mid x = uv, y = vw, v \neq \varepsilon\}$$

The outfix guided insertion of a string y into x is defined as

$$x \leftarrow y = \{x_1 uzv x_2 \mid x = x_1 uv x_2, y = uzv, u, v \neq \varepsilon\}.$$

We say that the nonempty substrings u and v are matched parts. The matched parts form a non-trivial outfix of y.

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We say that the nonempty substrings u and v are matched parts. The matched parts form a non-trivial outfix of y.
We can extend this operation for languages by setting

$$L_1 \leftarrow L_2 = \bigcup_{x \in L_1, y \in L_2} x \leftarrow y.$$

Outfix-guided insertion is not associative.

$$acd \leftarrow abc \leftarrow abcd$$

For a language L, define

- $\mathbb{OGI}^{(0)}(L) = L$,
- $\mathbb{OGI}^{(i+1)}(L) = \mathbb{OGI}^{(i)}(L) \leftarrow \mathbb{OGI}^{(i)}(L)$,

The outfix-guided insertion closure of L is

$$\mathbb{OGI}^{*}(L) = \bigcup_{i=0}^{\infty} \mathbb{OGI}^{(i)}(L).$$

Note that by selecting the entire string x as an outfix, we have $x \in x \leftarrow x$ for all $x \in \Sigma^*$ with $|x| \geq 2$.

Note that by selecting the entire string x as an outfix, we have $x \in x \leftarrow x$ for all $x \in \Sigma^*$ with $|x| \geq 2$.
This implies that for any language L,

$$L \setminus (\Sigma \cup \{\varepsilon\}) \subseteq \mathbb{OGI}^{(1)}(L)$$

and thus, $\mathbb{OGI}^{(i)}(L) \subseteq \mathbb{OGI}^{(i+1)}(L)$ for all $i \geq 1$.

Let L_1 and L_2 be languages. The right one-sided iterated insertion of L_2 into L_1 is defined by setting

- $\mathbb{ROGI}^{(0)}(L_1, L_2) = L_2$,
- $\mathbb{ROGI}^{(i+1)}(L_1, L_2) = L_1 \leftarrow \mathbb{ROGI}^{(i)}(L_1, L_2)$.

The right one-sided insertion closure of L_2 into L_1 is

$$\mathbb{ROGI}^*(L_1, L_2) = \bigcup_{i=0}^{\infty} \mathbb{ROGI}^{(i)}(L_1, L_2).$$

Let L_1 and L_2 be languages. The left one-sided iterated insertion of L_2 into L_1 is defined by setting

- $\mathbb{LOGI}^{(0)}(L_1, L_2) = L_1$,
- $\mathbb{LOGI}^{(i+1)}(L_1, L_2) = \mathbb{LOGI}^{(i)}(L_1, L_2) \leftarrow L_2$.

The left one-sided insertion closure of L_2 into L_1 is

$$\mathbb{LOGI}^{*}(L_1, L_2) = \bigcup_{i=0}^{\infty} \mathbb{LOGI}^{(i)}(L_1, L_2).$$

Let $L_1 = \{aacc\}, L_2 = \{abc\}$.

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$$\mathbb{ROGI}^*(L_1, L_2) = a^+bc^+$$

$$\mathbb{LOGI}^*(L_1, L_2) = \{aabcc, aacc\}$$

Proposition

If L_1 and L_2 are regular, then so is $L_1 \leftarrow L_2$.

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Construct an NFA with state set

$$Q \times (P \cup \overline{P} \cup \{\clubsuit, \heartsuit\}) \cup \overline{Q} \times P.$$

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x_1	u	z	v	x_2
$Q \times \clubsuit$	$Q \times P$	$\overline{Q} \times P$	$Q \times \overline{P}$	$Q \times \heartsuit$

Theorem

There exists a finite langauge L such that $\mathbb{OGI}^*(L)$ is nonregular.

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$$L = \{\$ a_3 a_1 b_1 b_3 \$,\ a_3 a_1 a_2 b_1,\ a_2 b_2 b_1 b_3,\ a_1 a_2 a_3 b_2,\ a_3 b_3 b_2 b_1,\ a_2 a_3 a_1 b_3,\ a_1 b_1 b_3 b_2\}.$$

$$L = \{a_3 a_1 a_2 b_1, a_2 b_2 b_1 b_3, a_1 a_2 a_3 b_2, a_3 b_3 b_2 b_1, a_2 a_3 a_1 b_3, a_1 b_1 b_3 b_2\}$$

$$\Downarrow$$

$$\$ a_3 a_1 b_1 b_3 \$$$

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$$a_3\, b_3\, b_2\, b_1,\ a_2\, a_3\, a_1\, b_3,\ a_1\, b_1\, b_3\, b_2\}$$

$$\Downarrow$$

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$$\Downarrow$$

$$\$ a_3 a_1 a_2 a_3 a_1 b_1 b_3 b_2 b_1 b_3 \$$$

$$\mathbb{OGI}^*(L) = \{\$ a_3 (a_1 a_2 a_3)^i z (b_3 b_2 b_1)^i b_3 \$ \mid i \geq 0, z \in S\}$$

$$S = \{a_1 b_1, a_1 a_2 b_1, a_1 a_2 b_2 b_a, a_1 a_2 a_3 b_2 b_1, a_1 a_2 a_3 b_3 b_2 b_1, a_1 a_2 a_3 a_1 b_3 b_2 b_1\}$$

Theorem

The outfix-guided insertion closure of a unary regular language is always regular.

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The 2-overlap catenation of x and y, denoted $x \overline{\odot}^2 y$ is defined as the set

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- If $x, y \in a^*$, then $x \leftarrow y = x \overline{\odot}^2 y$.
- If L is a unary language, then $\mathbb{OGI}^*(L) = 2\mathbb{OC}^*(L)$.
- The 2-overlap catenation closure of a regular language is regular.

Proposition

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For $L_1 = \{acdb, cabd\}$ and $L_2 = \{a\$b\}$, we have

$$\mathbb{ROGI}^*(L_1, L_2) = \{(ca)^i\$(bd)^i \mid i \geq 0\} \cup \{a(ca)^i\$(bd)^i b \mid i \geq 0\}$$

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For $L_3 = \{\$a_3 a_1 b_1 b_3\$\}$ and

$$L_4 = \{a_3 a_1 a_2 b_1, a_2 b_2 b_1 b_3, a_1 a_2 a_3 b_2, a_3 b_3 b_2 b_1, a_2 a_3 a_1 b_3, a_1 b_1 b_3 b_2\},$$

we have the same language as in the regular language case.

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$$L = \{\$a^n\$\$c^n \mid n \geq 1\} \cup \{\$a^n\$b^n\$ \mid n \geq 1\}$$

$$(L \leftarrow L) \cap \$a^+\$b^+\$c^+ = \{\$a^n\$b^n\$c^n \mid n \geq 1\}$$

Theorem

If L_1 is context-free and L_2 is regular, then $L_1 \leftarrow L_2$ and $L_2 \leftarrow L_1$ are context-free.

The same idea as for the case of regular L_1 and L_2 with the addition of stack operations for the context-free language.

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For $L_1 = \{cda^ib^ia^j \mid i,j \geq 1\} \cup \{ca^ib^ja^j \mid i,j \geq 1\}$ and $L_2 = \{cda\}$, we have

$$L_1 \leftarrow L_2 = cd \cdot (\{a^ib^ia^j \mid i,j \geq 1\} \cup \{a^ib^ja^j \mid i,j \geq 1\}).$$

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$$L_1 \leftarrow L_2 = cd \cdot (\{a^i b^i a^j \mid i, j \geq 1\} \cup \{a^i b^j a^j \mid i, j \geq 1\}).$$

For $L_3 = (a^* bac) + (aba^*)$ and $L_4 = \{b^j a^j c \mid j \geq 1\} \cup \{a^i b^i a^2 \mid i \geq 1\}$, we have

$$L_3 \leftarrow L_4 = \{a^i b^j a^j c \mid i, j \geq 1\} \cup \{a^i b^i a^j \mid i \geq 1, j \geq 2\}.$$

We say that a language L is closed under outfix-guided insertion if outfix-guided insertion of strings of L into L does not produce strings outside of L. That is, $(L \leftarrow L) \subseteq L$.

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There is a polynomial time algorithm to decide whether for a given DFA A the language $L(A)$ is og-closed.

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- Construct NFA B for $L(A) \leftarrow L(A)$.
- Let A' be the DFA obtained from A by interchanging final and non-final states.
- $L(B) \subseteq L(A)$ if and only if $L(B) \cap L(A') = \emptyset$.

Theorem

For a given context-free language L, the question of whether or not L is og-closed is undecidable.

- Via a PCP instance.

- Outfix-guided insertion of two regular languages is regular.
- There exist outfix-guided closures of finite languages that are non-regular.
- Outfix-guided insertion of two context-free languages may be non-context-free.
- Outfix-guided insertion of a context-free language and regular language is context-free.
- Outfix-guided insertion of a deterministic context-free language and regular language is not deterministic context-free.
- Deciding outfix-guided closure for a regular language is decidable and can be computed in polynomial time if given as a DFA.

Some open problems:

- Does there exist a regular language L such that the outfix-guided insertion closure of L is not context-free?
- If L is context-free, is $\mathbb{OGI}^*(L)$ context-sensitive?
- What is the complexity of deciding outfix-guided closure for a language given an NFA?