Math 152, Fall 2007, Week 1

1. Sketch the region R defined by $0 \le y \le 1/x^3$, $1 \le x \le 2$.

a) Find (exactly) the number a such that the line $x = a$ divides R into two parts of equal area.

b) Then find (to 3 places) the number b such that the line $y = b$ divides R into two parts of equal area.

2. Which has more area, the region in the first quadrant enclosed by the line $x + y = 1$ and the circle $x^2 + y^2 = 1$, or the region in the first quadrant enclosed by the line $x + y = 1$ and the curve $\sqrt{x} + \sqrt{y} = 1$? Justify your answer.

3. Let R be the parabolic region in the x-y plane bounded below by the curve $y = x^2$ and above by the line $y = 1$.

a) Sketch R. Set up and evaluate an integral that gives the area of R.

b) Suppose a solid has base R and the cross-sections of the solid perpendicular to the y-axis are squares. Sketch the solid and find its volume.

c) Suppose a solid has base R and the cross-sections of the solid perpendicular to the y-axis are equilateral triangles. Sketch the solid and find its volume.

4. Start with the region A in the first quadrant enclosed by the x-axis and the parabola $y = 2x(2 - x)$. Then obtain solids of revolution S_1, S_2, and S_3 by revolving A about the lines

$$y = 4, \qquad y = -2, \qquad \text{and} \qquad x = 4$$

respectively. All three solids are (unusual) "doughnuts" which are 8 units across, whose hole is 4 units across, and whose height is 2 units. Sketch them.

(a) Which do you expect to have larger volume, S_1 or S_2? Compute their volumes exactly and check your guess.

(b) Compute the volume of S_3. (It may be harder to guess in advance how S_3 compares in volume to S_2 and S_1.)