Math 152, Fall 2007, Week 9

1. If n is sufficiently large, the following functions of n fall in a definite increasing order, so that each function is very much larger than the one that precedes it. List the functions below in order of size from smallest to largest.

$$n, \quad n^n, \quad \ln n, \quad 4^n, \quad 2^n, \quad n \ln n, \quad 2^{n^2}, \quad \sqrt{n^6+1}, \quad (n^3+1)^{2/3}$$

Where would $n!$ fit in this list? Explain.

2. Under the hypotheses of the integral test, if $a_n = f(n)$ and $s_n = a_1 + a_2 + \cdots + a_n$, then

$$\int_1^n f(x)\,dx \le s_n \le a_1 + \int_1^n f(x)\,dx \qquad \text{for each positive integer } n.$$

In the case of the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$, this means that

$$\ln n \le 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \le 1 + \ln n \qquad \text{for each positive integer } n.$$

a) Find the analogous inequalities for the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ and for the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.
b) Estimate the sum of the first 10^{10} terms of the series, in each case. Then estimate the sum of the first 10^{100} terms.
c) Of the three series, which diverges the fastest? the slowest?

3. Use the comparison or limit comparison test to decide if the following series converge.

$$\sum_{n=1}^{\infty} \frac{4 - \sin n}{n^2 + 1} \qquad \sum_{n=1}^{\infty} \frac{4 - \sin n}{2^n + 1} \qquad \sum_{n=1}^{\infty} \frac{\sqrt{n+3}}{\sqrt{n^3+2}}$$

For the series which converge(s), give an approximation of its sum, together with an error estimate, as follows. First calculate the sum s_5 of the first 5 terms, Then estimate the "tail" $\sum_{n=6}^{\infty} a_n$ by comparing it with an appropriate improper integral or geometric series.

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