Math 152, Fall 2007, Week 10

1. The series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \qquad \text{and} \qquad \sum_{n=1}^{\infty} \frac{1}{n2^n}$$

both converge (why?). By coincidence it turns out that their sums are both equal to $\ln 2$. (We shall be able to understand this coincidence when we study Taylor series.)

Which series converges "faster" (and so numerically gives a more efficient way to get a numerical approximation for $\ln 2$)? Justify your answer by computing how many terms of each series must be added up to approximate $\ln 2$ with maximum allowed error of (a) 10^{-4}; (b) 10^{-6}.

2. In the following series x is a real number. In each case use the ratio test to determine all values of x for which the series converges absolutely. Analyze the behavior of the series at the endpoints in order to determine the interval of convergence.

$$\text{(a)} \sum_{n=0}^{\infty} \frac{nx^n}{n^2+1} \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{n^2(x-1)^n}{2^n} \qquad \text{(c)} \sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$$

3. Can you cook up a power series whose interval of convergence is the interval $(0, 1]$, that is, the interval defined by $0 < x \leq 1$? How about $(0, \infty)$? Give an explicit series or explain why you can't.

4. Consider an infinite series of the form

$$\pm 3 \pm 1 \pm \frac{1}{3} \pm \frac{1}{9} \pm \frac{1}{27} \pm \cdots \pm \frac{1}{3^n} \pm \cdots.$$

The numbers 3, 1, etc., are given but *you* will decide what the signs should be.

a) Can you choose the signs to make the series diverge?

b) Can you choose the signs to make the series sum to 3.5?

c) Can you choose the signs to make the series sum to 2.25?

In each case, if your answer is "Yes", then specify how to choose the signs; if your answer is "No", then explain.