

Applications of Regime-switching Forecasts to Predictive Regressions of the Equity Premium*

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Abstract

Recent papers have criticized the out of sample performance of predictive regression forecasts of the equity premium. Campbell and Thompson have proposed various sensible and economically appealing conditional strategies which improve the performance of these forecasts by modifying the forecast based on the signs of the regression results. This paper continues this line of research into modifications of the basic historical mean forecasts and the predictive regressions forecast. We focus on forecast strategies which utilize two regimes, where the time periods are classified by an indicator variable (which depends on either the past quarter's real GDP growth or the current equity premium), and which require the forecast to be nonnegative (this restriction was originally proposed by Campbell and Thompson). We find that when we use either the T-bill rate or the long term yield as the predictive regressor, these augmented forecast frameworks produce out of sample R_{os}^2 (relative to the ex-ante historical mean forecast) close to 1%, and in some cases, in excess of 1%. (This compares to R_{os}^2 values of 0.3% in the Campbell and Thompson forecast strategies, and they found that this was economically meaningful for portfolio allocation).

The purpose of economics is, to a great extent, a practical one: to enable people to **forecast** or influence economic activity.

Frederik Zeuthen, *Economic Theory*, 1955, page 5.

1 Introduction

Recent research has explored the poor out-of-sample results obtained by predictive regression forecasts of the equity premium. Goyal and Welch (2004, [22]) demonstrated the poor out-of-sample performance of many predictive variables, and stated that historical average returns almost always generate better equity premium forecasts:

*This version is preliminary; please do not distribute or cite. We thank Walter Torous, Robert Weiss (from biostatistics), Albert Sheen, and Ted Shieh for helpful discussions, questions, and ideas, and we thank Samuel Thompson for generously providing his data set. We remain responsible for all remaining errors.

Our paper has systematically investigated the empirical real-world out-of-sample performance of plain linear regressions to predict the equity premium. We find that none of the popular variables has worked—and not only post-1990. In our monthly tests, we can solidly reject regression model stability for all variables we examined, even though we use the CUSUMSQ test which is known to be fairly weak. For successful out-of-sample prediction, we will either need a different technique or a different variable. ... Instead our paper suggests only that our profession has yet to find a variable that has had meaningful robust empirical equity premium forecasting power, at least from the perspective of a real-world investor. ([22], p.19-20)

Goyal and Welch emphasized the importance of evaluating the performance of predictive regressions by analyzing the out-of-sample forecasts using only the knowledge that could be known ex-ante as of the forecast date.

Campbell and Thompson (2005, [11]) responded to the Goyal-Welch critique. They considered several variables in addition to those considered by Goyal and Welch. First, they used total returns (including dividends) rather than the price returns of Goyal and Welch, and they obtained similar negative results about predictability. Secondly, they argued that real-world investors would impose some restrictions on the signs of the regression coefficients (in particular the slope coefficient) as well as a restriction on the sign of the forecasted value of the equity premium (namely that the forecast be non-negative). They claimed that these are sensible and intuitive constraints on the forecasts, and they defined 4 strategies based on these constraints, as follows:

1. panel A uses the predictive regressions in a straightforward way.
2. panel B implements the following restriction on the forecast: if the slope is negative (except for the variables inflation and the default yield spread), then the forecast defaults to the ex-ante historical average (calculated using only the equity premia up to that point). (For inflation and the default yield spread, the expected slope is negative, so if the slope is positive, the forecast defaults to the ex-ante historical average).

3. panel C restricts the forecast to be non-negative; if the regressions yield a negative forecast, then the forecast is set to 0. Note that for investors who can not short sell the stock market and who makes allocation decisions based on the forecast, this restriction might be redundant; non-positive forecasts of the equity premium would induce risk-averse investors to avoid the stock market entirely and instead only own the risk free security.
4. panel D imposes first the panel B restriction on the sign of the slope coefficient and then the panel C restriction on the sign of the forecast itself.

In some sense, we may view strategies B, C, and D as ways of imposing absolute Bayesian priors on the results of the regressions; if the signs of computed values are contrary to the priors, the forecast is set either to zero or to the historical mean (which represents a judgment that the information content of the regressor is worthless).

Campbell and Thompson consider a myopic log-utility (and more generally, an investor with constant relative risk aversion γ) investor who makes an allocation decision between the T-bill rate and the stock market every month. The third contribution of the Campbell and Thompson paper is the observation that although the out of sample R^2 values of the forecasts have very small absolute magnitudes (0.100% to 0.476%), a CRRA utility investor (in the above setting) can still achieve a significant portfolio performance improvement by using these forecasts. As a matter of comparison, they state that a myopic log-utility investor utilizing the smoothed earnings-price ratio (with an out of sample R^2 of 0.309%) could increase the monthly portfolio return by 30 basis points or 3.6% annualized.¹

¹We may view this as a discrete time version of the allocation strategy discovered by Merton for CRRA investors in the continuous time setting; the fraction of wealth allocated to the risk asset is $(\mu - r)/(\gamma\sigma^2)$, where $(\mu - r)$ is the equity premium and σ^2 is the volatility. Campbell and Thompson argue that an investor who uses a predictor with even a small positive R_{os}^2 will see a decrease in the perceived risk σ^2 and hence will increase the allocation fraction to the risky asset.

There are at least two obvious ways that the simple predictive linear regression model could be misspecified:

1. non-linear nonstationarities in the underlying time series. This could be approached by the development of general one-to-one series statistical transformations of the underlying time series which convert them to stationary time series.
2. regime-switching in the underlying linear relationships between the predictive variable and the equity premium.

This paper will analyze some ways to address the second possibility, with the aim of improving the performance of the forecasts. We find that for the post 1957 period, the interest rate variables, the T-bill rate and the long term yield are significantly improved by the use of threshold regimes, after we impose a nonnegativity constraint (the same as the panel C constraint above).

1.1 Preliminary comments on the data and on mathematical notation

In terms of data and notation, we follow the two papers Goyal and Welch (2004, [22]) and Campbell and Thompson (2004, [10]) closely.

1. We define the terms *in sample* and *out of sample* by following the convention of the two papers; we use the term *in sample* to describe the statistics and regressions which use the entire sample period. We use the term *out of sample forecasts* (and more generally *out of sample*) to denote the series of rolling one-period ahead predictive regressions.
2. We use the same set of predictor variables as in Campbell and Thompson (2004, [10])². They used many of the same variables as those in the Goyal and Welch paper, with a few changes.³

²We thank Samuel Thompson for generously providing the data set analyzed in the paper

³The data in raw form is available on Amit Goyal's website with some additional earlier data available from Robert Schiller's website.

Table 1 is virtually identical (up to numerical rounding) to the first table in their paper, with one addition. We note that the original forecasts using inflation as a predictive regressor were infeasible because the economic data was actually released 2 weeks later. In particular the measure of inflation at time t is the ratio of the CPI (consumer price index, all urban consumers) at time t and the CPI at time $t - 1$ minus 1. The CPI at the end of a month is actually released 15 days afterwards. The original calculation is now labelled *inflation (infeasible)*; the new calculation (with the inflation data lagged by one month) is included as the last variable *inflation adjusted*.

3. In defining the equity premium, we follow Campbell and Thompson (2004, [10]). Define the total return of the S&P 500 at time $t + 1$, $1 + sp_{t:t+1}$, as the sum of the value weighted index at time $t + 1$ together with the dividends, all divided by the time t index. Let $R_{f;t:t+1}$ denote the risk free rate from time t to time $t + 1$ (and this is known at time t). The equity premium at time $t + 1$ is the difference in the log total return and the log return of the risk-free rate⁴;

$$p_{t+1} \equiv \log(1 + sp_{t:t+1}) - \log(1 + R_{f;t:t+1}).$$

Goyal and Welch (2003, [21]) advocate the use of graphs which plot the difference in the sum of squared forecast errors between the historical mean and the predictive regression; these graphs demonstrate that the out of sample predictability of the book to market variable is almost entirely due to the years 1973 and 1974. The performance of the book to market predictive regression

⁴We note that this is one of the changes from Campbell and Thompson (2004, [10]) and the 2005 version ([11]). They switched from using log returns in the 2004 version to using simple returns in the later version; in the later version, they write

All the regressions we have reported predict simple stock returns rather than log stock returns. The use of simple returns makes little difference to the comparison of predictive regressions with historical mean forecasts, but all forecasts tend to generate higher mean residuals when log returns are used. The reason for this is that high stock market volatility in the 1920's and 1930's depressed log returns relative to simple returns in this period. Thus the gap between average stock returns in the late 20th Century and the early 20th Century is greater when log returns are used.

In this paper, we only consider log returns.

declined almost every year before and after the early 1970's. We will present such graphs whenever applicable to test whether any positive performance is due to anomalous time periods.

Finally, for a given time series $\{y_t\}$, we will write \hat{y}_{t+1} to denote a time t forecast for y_{t+1} . The historical mean forecast can then be written by the equation $\hat{y}_{t+1} = \bar{y}_{t+1} (= (\sum_{k=0}^t y_k)/(t+1))$

1.2 Definition of the out of sample R^2 statistic

Campbell and Thompson ([11], pages 5-6) defined R_{os}^2 , an out-of-sample R^2 statistic which is comparable with the in-sample R^2 statistic. This compares the mean squared errors of a forecast with the historical mean forecast on a relative basis:

$$R_{os}^2 \equiv 1 - \frac{\sum_{t=1}^T (r_t - \hat{r}_t)^2}{\sum_{t=1}^T (r_t - \bar{r}_t)^2}$$

In this equation, our time series is $\{r_t\}_{-s \leq t \leq T}$, where $s \geq 0$ and $\{r_{-s}, \dots, r_0\}$ is the *training period*; \hat{r}_t is the extrapolated value from the predictive regression with data from time $-s$ up to time $t-1$, and $\bar{r}_t = (\sum_{k=-s}^{t-1} r_k)/(t+s)$ is the historical mean from up to time t . (Our convention is that if $s=0$, then our predicted value for time 1 equals the time 0 value; $\hat{r}_1 = r_0$). The sign of R_{os}^2 indicates the relative performance of $\{\hat{r}_t\}$ and $\{\bar{r}_t\}$ on a RMSE (root mean square error) basis; a positive R_{os}^2 occurs exactly when the RMSE of the predictor is less than the RMSE of the historical mean forecast.

2 Regime switching

Why should we study regime switching models to forecast the equity premium? There are a number of reasons:

1. Economic intuition suggests that the equity premium might be different for high growth

and low growth states of the world. Indeed, the original paper on the equity premium puzzle, Mehra and Prescott (1985, [31]), considered a multiple state world with varying equity premia across states. Although this intuition alone might help to provide ex-post stories to explain past equity premia, it is not obvious that there is either a good enough predictor of the next period state of the world, or that the state of the world is persistent enough for use in multiple regime predictive linear regression forecasts. In fact, if the relationship is weak enough, the classification might well decrease forecasting ability, because it will reduce the number of data points per regime and hence will increase the parameter estimation error from the least squares regression.

2. There is an extensive literature of research into regime switching forecast models of time series and economic data. We mention two recent papers: Clements and Krolzig (2004, [12]) studied the ability of regime switching models to replicate stylized facts about US macroeconomic data, and van Dijk and Franses (2003, [35]) study two-regime threshold linear regressions, which can be specified as

$$y_{t+1} = (\phi_{1,0} + \phi_{1,1}x_t)I[z_t > c_t] + (\phi_{0,0} + \phi_{0,1}x_t)(1 - I[z_t \leq c_t]) + \eta_t \quad (1)$$

Here, z_t is the threshold variable, c_t is our threshold value at time t , η_t is the noise term, and $I[A]$ is the indicator function of an event A which takes the value 1 if A occurs and takes the value 0 otherwise. (van Dijk and Franses ran Monte Carlo simulations of an AR(1) process, and considered forecasts based on threshold autoregressions. In their setting, $x_t \equiv y_t$, $z_t = y_t$, and the threshold value $c_t = c$ is a constant. We have altered the subscripts of ϕ and switched the inequalities in the indicator).

The Campbell and Thompson portfolio strategies may be viewed as implementations of regime switching. They state

A regression estimated over a short sample period can easily generate perverse results, such as a negative coefficient when theory suggests that the coefficient should be positive. Since out-of-sample forecast evaluation begins as little as 20 years after the start of the data set, this can be an important problem in practice. To illustrate this point, Figure 1 plots the rolling regression coefficients of total returns and price returns on the dividend-price ratio from 1927 through 2003, showing that the coefficients were negative until the end of World War II. In practice, an investor would not use a perverse coefficient but would likely conclude that the coefficient is zero, in effect imposing prior knowledge on the output of the regression. ([11], p.6)

Implicitly, panel B strategies implement 2 states for the value of the forecast (the regression calculation and the ex-ante historical mean), the panel C strategies implement 2 states (the regression calculation and zero), and the panel D strategies implement 3 states (the regression calculation, the ex-ante historical mean, and zero). Observe that in panels B and D, if the slope has a “perverse” sign, then the forecast disregards the regression value entirely.

We will study models in which the strategy does not discard information, but instead uses additional information to classify time periods into 2 regimes. We study models that conform to the specification of equation (1). In all of our models, the predictand y_{t+1} will be the next period equity premium, and the models will study different choices of the predictor x_t , the threshold variable z_t , and the threshold values c_t . In keeping with the requirement that the forecast strategy be real-time and only implement information known up to and including time t to forecast the time $t + 1$ premium, we will usually specify the threshold value c_t as the historical mean of the threshold variable up to that point: $c_t = \bar{z}_t (= (\sum_{k=0}^{t-1} z_k)/t)$.⁵

In section 2.2, our threshold variable will be the past quarter’s GDP growth (suitably lagged by one month), and in section 2.3, our threshold variable will be the current month’s equity premium

⁵Our convention is that c_0 will be either 0 or 1, depending if z_t is a gross or net variable. Here z_t could be either a return or a growth rate.

(p_t , if we are trying to predict p_{t+1}).

2.1 (Statistical tests) Testing for structural breaks in the regression forecasts

Structural breaks are sudden and abrupt changes in the underlying properties of a time series; typically they involve changes in mean and variance, but other properties can also be analyzed. There have been many finance papers which study structural breaks in the equity premia time series in isolation, separate from any linear predictive regressions. Pastor and Stambaugh (2001, [32]) utilize a Bayesian approach which assumes a positive relationship between the equity premium and volatility across structural regimes and which also assumes that the excess returns are i.i.d normal within regimes. They conclude that there were breaks around 1928, 1941, and 1991 within a 1834 to 1996 framework. Kim, Morley, and Nelson (2005, [26]) weaken the assumptions and allow for temporary changes in the volatility and the mean of the equity premium, and find (under more stringent criteria) that there was only one structural break in 1941 for the time period 1926 to 1999.

They state

The analysis favors a model that relates the equity premium to Markov-switching changes in the level of market volatility and accommodates volatility feedback. For this model, there is evidence of a one-time structural break in the equity premium in the 1940s, with no evidence of additional breaks in the postwar period. The break in the 1940s corresponds to a permanent reduction in the general level of stock market volatility. Meanwhile there appears to be no change in the underlying risk preferences relating the equity premium to market volatility. The estimated unconditional equity premium drops from an annualized 12% before the break to 9% after the break.

Hence there are frameworks which conclude that the equity premia time series possesses at least one structural break. This suggests that linear predictive regressions of the equity premium might also have structural breaks, and we try to apply some of the recently discovered tests for structural breaks in the predictive regressions. Andrews (1993, [2]) provided a test for sudden one time changes in linear regression parameters, based on the Chow test and F statistics. Bai (1997,

[4]) and Bai and Perron (2003, [5]) developed computational tests to detect and identify multiple break points. In these tests, the null hypothesis of a single linear regression regime is compared with the specified alternative of 2 or more disjoint regimes of the regressor variables with distinct linear relationships for each regime.

Another class of linear regression structural change tests arise from the generalized fluctuation test framework. Leisch, Hornik, and Kuan (2000, [28]) is an example. These tests analyze the limiting asymptotic distribution of the residuals arising from the linear regressions, and provide tests to detect more general deviations from the null hypothesis of a single linear regression regime; the alternative is not specified. These tests are very interesting from the perspective of trying to improve predictive regressions; they provide statistical tests and techniques to monitor a time series in real time, to attempt to detect structural breaks as soon as possible after they occur.

The ex-post tests for structural breaks find evidence of a structural break in the predictive regressions in the 1920's, especially for the market based variables. Unfortunately, the monitoring results to detect structural breaks in real time are heavily parameter dependent, and hence they would have been of limited usefulness to a real-time investor. Structural breaks were found in real time for the stock market based variables but rarely for the macroeconomic variables. For example, the moving sum of residuals test, which adds the rolling regression residuals over a bandwidth of past values depends on a bandwidth or fraction of the entire history of the time series; the asymptotic distribution (suitably scaled) is that of the increments of a Brownian motion. The tests using a bandwidth equal to one-fourth of the past history detects breakpoints (at the 5% level) in the linear coefficients of the lagged dividend to price ratio in 1946, 1998, and 2001, whereas the bandwidth choice of one-half the past history detects breakpoints for the same variable in 1959, 1961, 1962, and 1966. Given the parameter dependence of these results, the analysis of these tools

for real-time portfolio selection will require some insight as to why certain parameter values are higher than others.

2.2 (Macroeconomic variables) Using GDP growth as a threshold variable

Faugère and Van Erlich (2003, [16]) analyzed the long-run equity premium in the context of GDP growth; they write

In this paper, we provide two alternative explanations for the long-run equity premium. One is based on GDP growth; the other is based on a portfolio insurance motive. We derive the long-run ex-ante equity premium and long run corporate debt return using a supply-side growth model. Our model accurately replicates the arithmetic average historical returns for the S&P 500 and the relatively riskless T-Bill. Our first conclusion is that the equity premium defined as the difference between the S&P 500 stock return and 3-month government T-Bill can be fully explained by GDP growth in the long run. One key reason is that equity is a claim to earnings and dividend growth while fixed income instruments are just that. ([16], p. 18)

This leaves open the question about quarterly real GDP growth and monthly predictive regressions. We now examine the usefulness of quarterly real GDP growth as a threshold variable for predictive regressions. There are several reasons why the use of the previous quarter's GDP growth might not work effectively in the prediction of the equity premium. First, the information is lagged by either 1 month (for advance estimates) or 3 months (for the final estimates) (see the footnote for more details). For certain choices of predictor variables, most of the additional information from

⁵This section presents preliminary results. It presents calculations on forecasts based on the 2005 estimates of the real quarterly GDP time series, obtained from the FRED II database at the St. Louis branch of the Federal Reserve (<http://research.stlouisfed.org/fred2/>). (Specifically, the real quarterly GDP data was read from <http://research.stlouisfed.org/fred2/series/GDPC96/downloaddata/GDPC96.txt>). However, these results are preliminary in that a real-time investor would not have known the 2005 estimates; they would only have known the advance GDP estimates released one month after the end of the quarter. (For example, the advance estimate of the end of December GDP is released at the end of January. We do lag the threshold variable by one month to reflect this delay in the release of information). In particular, the findings in this section are subject to the change that results from using the advance estimates as opposed to the final revised estimates (which are released 3 months after the end of the quarter; for example, the end of December GDP final estimate is released at the end of March). A more realistic simulation would use the advance estimates rather than the final estimates. Recently we became aware of the *Real Time Data Set for Macroeconomists* provided by the Philadelphia branch of the Federal Reserve. (It is available at <http://www.phil.frb.org/econ/forecast/reaindex.html>). This data set contains snapshots of announced economic data; in particular it contains the real-time advance GDP estimates, free of any future revisions. We are working to rerun the calculations utilizing these real-time advance GDP estimates rather than the 2005 numbers. Since we only use the GDP number as a threshold variable (namely we only compare it to the ex-ante historical mean of the GDP time series), we are hopeful that the results of this section will not change dramatically.

the regime switch could have been impounded into prices within that lag period, and in this case, the reduction in data points per regime (having gone from 1 regime to 2) with its resultant increase in parameter estimation error could in fact decrease forecast precision and hence accuracy. Second, the quarterly nature of this indicator poses obvious disadvantages for a threshold variable that is updated monthly; if real GDP growth has enough monthly volatility, then forecast accuracy for the regime switching forecast strategy will decrease even if there is a link between real GDP growth and the equity premium.

We simulate such a strategy. We start the time series in May 1947 (the first date that real GDP data is available), and we start the forecasts (for both the historical mean predictor and our predictive regressions) in January 1957, to give a “training period” of nearly 10 years. Table 2 presents the summary results. We note that the switch to a 2 regime setting uniformly worsens all the predictive regressions; there are none with a positive R_{os}^2 .

Next, we impose the panel C constraint, the nonnegativity constraint on the forecasts; Table 3 presents the summary results. We note the R_{os}^2 results for the two best predictors, the T-bill rate and the long term yield, 1.14% and 0.71% respectively. Another curious feature is the contrast in the t -stats among the variables; the two best predictors have the highest t -stats (in absolute value) for the high growth periods, whereas they have unremarkable t -stats for the low growth periods.

2.3 (Market data) Using past equity premia

Finally, we consider a 2-regime strategy that uses the current month’s equity premium p_t as the threshold variable. The threshold value is the historical mean of the the equity premium \bar{p}_t . We find qualitative results similar to the previous section; without the nonnegativity restriction, the 2 regime setting uniformly worsens all the predictive regressions, Table 4 presents the results.

Once again, we impose the panel C constraint; we impose the usual nonnegativity constraint;

Table 5 presents the results. As before, the two best predictors are the T-bill rate and the long term yield; they have R_{os}^2 values of 1.03% and 0.50% respectively. The in-sample t -statistics make these strategies fairly suspect from an intuitive point of view; the t -statistics for the growth indicator versions of these variables are fairly negligible.

We observe that the t -stat for the T-bill rate in the low growth time periods is quite high; it is 3.21. The significance of the T-bill rate as a predictor during these low growth periods can be illustrated by the stylized facts that in a low growth environment (including recessions), high interest rates are quite detrimental to stock owners. This was the case for periods of time in the 1970's and the early 1980's. It is also the case that in a low growth period, a low interest rate can signal future high equity premiums; this can be seen in the case of the early 2000's.

The previous simulations used the same time period as our GDP growth indicator simulations; all time series started in 1947. For our next simulation, we use the same time periods as Campbell and Thompson; this allows comparison with their results. Table 6 presents the market regime switching results, and Table 7 presents the results after imposing the usual nonnegativity constraint.

3 Conclusion and Discussion

This paper analyzed several aspects of the predictive linear regressions of the equity premium. We observe the following:

1. One possibility to improve the regime switching strategies is to allow the optimum threshold value be calculated as the optimum percentile of the threshold variable. Our strategies simply used the mean, and if the distribution of the threshold variable is not very skewed, this would correspond to the median, or the 50th percentile. One direction for future research is to calculate the effects of allowing the strategy to calculate what the optimum percentile value

had been in the past and to use this in the future.

2. Keeping track of information release is critical when simulating investment strategies based on macroeconomic data. The infeasible inflation predictor provides exceptionally good predictive power. The large contrast between the infeasible and the adjusted versions of the time series of the inflation predictor variable raises some interesting questions, about the role of information about inflation on the equity premium (and on expectations of future values of the equity premium).
3. We compare Table 7 with Table 5. There are at least 2 noteworthy differences: the first three market variables (lagged dividend price ratio, lagged earnings price ratio, and smoothed earnings price ratio) do well in the longer sample (where data start from 1871 or 1881 and forecasts start in 1927), but they perform badly in the smaller sample, with forecasts starting from 1957. One possible hypothesis is that perhaps this predictability was exploited by traders. One fact that might have minor relevance is the 1941 structural break in the equity premium. (We mentioned the finding of Pastor and Stambaugh in an earlier section). This 1941 structural break might reduce the perceived predictability of stock market variables if the evaluation period spans years both before and after 1941. Another striking contrast between the two tables is the surprisingly good performance of the T-bill rate and the long term yield in the more restricted time frame, but the poor performance in the longer time frame. One reason for the poor performance of the T-bill forecast is the fact that the T-bill rate was **constant** between 1942:07 to 1947:06. Since obviously the equity premium varied during that time frame, any linear regression with the T-bill rate as a regressor and which includes this time frame will penalize the T-bill rate severely.
4. The finding that inflation (at least the infeasible variable) is a relatively good predictor in-

creases focus on the roles of TIPS (Treasury Inflation Protected Securities) as either providing a predictive role (through its price time series) or as hedges for stock investors against the risk of higher inflation. Can an investor use TIPS prices to obtain a higher frequency time series of inflation expectations, and use this to help predict future equity premia? Perhaps a strategy could be calculated, based on the prevailing TIPS yield, the implied expectation about future inflation and use this as a proxy for the infeasible inflation variable. During periods of increasing inflation, to what extent could TIPS serve as a useful hedge for existing stockholders against falling equity premia?

5. The restriction that forecasts are always nonnegative is almost always useful in improving the R_{os}^2 , and it occasionally produces a dramatic improvement in forecast accuracy. How specific is this improvement when forecasting equity premia in the US as opposed to other nations? Goetzmann and Jorion (1999, [19]) argue that the high levels of the equity premium in the US are subject to survivorship. If we were to apply the nonnegativity restriction when using linear predictive regressions for the equity premium in other countries, how much of a forecast advantage would remain? Also, are there any indicators which can reliably forecast negative equity premia in advance?
6. The best predictors tend to be those whose regressions underestimate the equity premium. During the October 1987 crash, those predictors received a substantial increase in R_{os}^2 and the difference in RMSE, whereas the predictors which overestimate the equity premium (in terms of average error) were penalized severely. Observe the $\bar{\varepsilon}_{os}$ column in Table 4; the only variables with a positive average error were the T-bill rate and the long term yield (except for the lagged dividend price ratio which had a negligible average error of 0.038% as compared to 0.450% and 0.423%). We conjecture that in general, risk-averse investors prefer predictors

which underestimate the equity premium over those which overestimate the equity premium (if other factors such as RMSE are equal). To take an extreme example, a severely risk averse investor might feel “safer” using an underestimating predictor because she feels less exposed or vulnerable to a crash. To what extent, if any, will these underestimating predictors provide greater utility gains? This might also be reflected in a preference for right-skewed distributions for the residual forecast errors, and this would not conform to the normality (or at least symmetry) assumption on the distribution of forecast errors assumed by the Merton and the Campbell-Thompson analysis. These underestimating regressors are also typically those who benefit most from the panel C restriction, the nonnegativity requirement on the forecast of the equity premium. Evidence for this can be seen by comparing the $\bar{\varepsilon}_{os}$ column in Table 5 with that column in Table 4; the nonnegativity constraint placed on the T-bill and long term yield forecast drastically reduced the average underestimate from 0.450% to 0.039% (for the T-bill forecast) and from 0.423% to 0.062%. The average error for the underestimating predictors improves dramatically with the restriction, as the most severe underestimates, those which forecast a negative value for the equity premium, become truncated to zero.

7. Regime switching based on macroeconomic variables or the current equity premium is somewhat useful in boosting forecast performance, for certain variables. Are there identifiable time series properties of the two best predictors, the T-bill rate and the long term yield, which help to distinguish them from the other predictor variables? This is particularly interesting because those two predictors experience a tremendous increase in forecasting power (at least as measured by the RMSE) whereas virtually every other variable considered suffers dramatic degradation in forecast performance. This is partly explained by October 1987, but not entirely; the slopes of the difference in the sum of the squared errors is consistently positive

for those two predictors, and it is consistently negative for others. Is this due entirely to the underestimation of their predictive forecasts?

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Predictive Regressors	In Sample		Out of Sample Forecast			
	t -stat $_{i,s}$	adjusted $R^2_{i,s}$	$\Delta RMSE_{os}$	$\bar{\epsilon}_{os}$	$\bar{\epsilon}_{os,hm}$	R^2_{os}
Market Variables						
lagged dividend price ratio	0.97978	-0.00251%	-0.01033%	0.23887%	0.15015%	-0.36449%
lagged earnings price ratio	2.33050	0.27740%	0.00750%	0.22131%	0.15015%	0.26396%
smoothed earnings price ratio	2.42605	0.33037%	-0.00183%	0.37420%	0.14791%	-0.06442%
dividend payout ratio	1.64400	0.10745%	-0.00415%	-0.01359%	0.15117%	-0.14617%
book to market ratio	2.12251	0.35209%	-0.01004%	0.06306%	0.01215%	-0.47523%
net equity expansion ratio	0.15245	-0.10605%	-0.00232%	0.01925%	0.02039%	-0.10994%
Macroeconomic Indicators						
tbill rate	1.63321	0.16563%	0.00436%	0.31562%	0.04353%	0.20133%
long term yield	1.10059	0.02075%	-0.01140%	0.36201%	0.01903%	-0.51695%
long term return	1.17053	0.03961%	-0.02513%	-0.06079%	-0.00749%	-1.18540%
term spread	1.45611	0.11134%	0.00600%	-0.04562%	0.04353%	0.27709%
default yield spread	-0.56097	-0.06737%	-0.00604%	0.04809%	0.01903%	-0.27375%
default return spread	0.98105	-0.00402%	-0.00896%	0.00053%	-0.00749%	-0.42193%
inflation (infeasible)	-1.69728	0.18459%	0.00979%	0.07138%	0.02733%	0.44350%
inflation adjusted	-0.83168	-0.03035%	-0.00148%	0.05629%	0.02752%	-0.06716%

Table 1: This table presents the results of the unmodified predictive regressions. It duplicates the first table in Campbell and Thompson (2004, [10]) closely, with the exception of one addition, the *inflation adjusted* variable. $\bar{\epsilon}_{os}$ is the average forecast error (the premium minus the forecast); a positive $\bar{\epsilon}_{os}$ means that on average the premium was greater than the forecast. $\bar{\epsilon}_{os,hm}$ is the average error for the ex-ante historical mean forecast.

Predictive Regressors	In Sample		Out of Sample Forecast			
	t -stat _{<i>t</i>,<i>s</i>} (growth)	t -stat _{<i>t</i>,<i>s</i>} (nongrowth)	adjusted $R^2_{t,s}$	$\Delta RMSE_{os}$	$\bar{\varepsilon}_{os}$	R^2_{os}
Market Variables						
lagged dividend price ratio	0.31127	2.77861	2.43701%	-0.00905%	0.07357%	-0.41714%
lagged earnings price ratio	0.46398	2.17450	2.02283%	-0.01322%	-0.10895%	-0.61011%
smoothed earnings price ratio	0.00701	2.69170	2.35481%	-0.01616%	-0.02599%	-0.74580%
dividend payout ratio	0.36005	-1.29787	1.56775%	-0.02734%	-0.19612%	-1.26361%
book to market ratio	-0.50844	1.53669	1.68533%	-0.03469%	-0.20451%	-1.60446%
net equity expansion ratio	-0.13046	0.03124	1.30467%	-0.03964%	-0.27038%	-1.83425%
Macroeconomic Indicators						
tbill rate	2.23769	1.53154	2.37028%	-0.02404%	0.49970%	-1.11062%
long term yield	1.86328	0.60084	1.86177%	-0.04103%	0.44923%	-1.89925%
long term return	0.50110	2.60565	2.32536%	-0.06137%	-0.36721%	-2.84730%
term spread	0.69184	2.12045	2.02734%	-0.02307%	-0.46103%	-1.06581%
default yield spread	0.54001	-1.80417	1.82020%	-0.03110%	-0.45612%	-1.43804%
default return spread	0.54684	0.15910	1.34964%	-0.04011%	-0.30287%	-1.85616%
inflation (infeasible)	-1.59807	-0.58634	1.72578%	-0.01250%	-0.22713%	-0.57684%
inflation adjusted	-1.75922	-1.46447	2.07428%	-0.00317%	-0.19731%	-0.14612%

Table 2: This table presents the simulation results for the regime switching strategy with the previous real quarterly GDP growth as the threshold variable. (Data starts 1947:05, forecasts begin 1957:01, and the simulation ends 2003:12). **Note:** $\bar{\varepsilon}_{os}$ is the average forecast error (the premium minus the forecast); a positive $\bar{\varepsilon}_{os}$ means that on average the premium was greater than the forecast. $\bar{\varepsilon}_{os,tm}$ is the average error for the ex-ante historical mean forecast. For these results, since all calculations began with the same month and year, $\varepsilon_{os,tm}$ was always the same for every variable; it was about -0.31364% .

Predictive Regressors	In Sample		Out of Sample Forecast			
	t -stat $_{i,t}$ (growth)	t -stat $_{i,t}$ (nongrowth)	adjusted $R^2_{i,t}$	$\Delta RMSE_{os}$	$\bar{\varepsilon}_{os}$	R^2_{os}
Market Variables						
lagged dividend price ratio	0.31127	2.77861	2.43701%	-0.00215%	-0.00694%	-0.09907%
lagged earnings price ratio	0.46398	2.17450	2.02283%	-0.01649%	-0.13072%	-0.76114%
smoothed earnings price ratio	0.00701	2.69170	2.35481%	-0.01116%	-0.08928%	-0.51499%
dividend payout ratio	0.36005	-1.29787	1.56775%	-0.01585%	-0.27485%	-0.73173%
book to market ratio	-0.50844	1.53669	1.68533%	-0.03204%	-0.20654%	-1.48145%
net equity expansion ratio	-0.13046	0.03124	1.30467%	-0.02609%	-0.28174%	-1.20535%
Macroeconomic Indicators						
tbill rate	2.23769	1.53154	2.37028%	0.02472%	0.11039%	1.13574%
long term yield	1.86328	0.60084	1.86177%	0.01542%	0.14555%	0.70892%
long term return	0.50110	2.60565	2.32536%	-0.05185%	-0.46912%	-2.40282%
term spread	0.69184	2.12045	2.02734%	-0.01967%	-0.56835%	-0.90828%
default yield spread	0.54001	-1.80417	1.82020%	-0.03088%	-0.45659%	-1.42774%
default return spread	0.54684	0.15910	1.34964%	-0.03554%	-0.31641%	-1.64383%
inflation (infeasible)	-1.59807	-0.58634	1.72578%	-0.00363%	-0.26338%	-0.16714%
inflation adjusted	-1.75922	-1.46447	2.07428%	0.00250%	-0.28000%	0.11489%

Table 3: This table presents the simulation results for the regime switching strategy with the previous real quarterly GDP growth as the threshold variable, along with a nonnegativity constraint on the forecast. (Data starts 1947:05, forecasts begin 1957:01, and the end of the simulation is 2003:12). **Note:** $\bar{\varepsilon}_{os}$ is the average forecast error (the premium minus the forecast); a positive $\bar{\varepsilon}_{os}$ means that on average the premium was greater than the forecast. $\bar{\varepsilon}_{os,tm}$ is the average error for the ex-ante historical mean forecast. For these results, since all calculations began with the same month and year, $\bar{\varepsilon}_{os,tm}$ was always the same for every variable; it was about -0.31364% .

Predictive Regressors	In Sample			Out of Sample Forecast			
	t -stat _{is} (growth)	t -stat _{is} (nongrowth)	adjusted R^2_{is}	$\Delta RMSE_{os}$	$\bar{\varepsilon}_{os}$	$\bar{\varepsilon}_{os,hm}$	R^2_{os}
Market Variables							
lagged dividend price ratio	1.67039	1.67478	2.08118%	-0.01475%	0.03845%	-0.31364%	-0.68083%
lagged earnings price ratio	1.70776	1.13730	1.88060%	-0.01619%	-0.14555%	-0.31364%	-0.74706%
smoothed earnings price ratio	1.73907	1.20137	1.91799%	-0.02469%	-0.06861%	-0.31364%	-1.14042%
dividend payout ratio	0.09985	-1.26622	1.50237%	-0.02468%	-0.24674%	-0.31364%	-1.13995%
book to market ratio	1.26290	0.08390	1.50072%	-0.03897%	-0.23296%	-0.31364%	-1.80343%
net equity expansion ratio	0.08034	0.57828	1.31597%	-0.04867%	-0.33105%	-0.31364%	-2.25470%
Macroeconomic Indicators							
tbill rate	0.46348	3.20709	2.78493%	-0.02123%	0.44981%	-0.31364%	-0.98037%
long term yield	0.15379	2.30193	2.04178%	-0.03587%	0.42337%	-0.31364%	-1.65923%
long term return	0.03198	3.53540	3.06896%	-0.02910%	-0.33682%	-0.31364%	-1.34489%
term spread	0.67755	2.24479	2.06718%	-0.03820%	-0.46394%	-0.31364%	-1.76731%
default yield spread	-1.04179	-0.15548	1.42866%	-0.03993%	-0.44546%	-0.31364%	-1.84794%
default return spread	0.22141	0.24979	1.28228%	-0.03681%	-0.31706%	-0.31364%	-1.70302%
inflation (infeasible)	-0.09332	-2.14668	1.93963%	-0.00635%	-0.24559%	-0.31364%	-0.29290%
inflation adjusted	-1.40588	-1.83601	2.05379%	-0.02126%	-0.23459%	-0.32396%	-0.98156%

Table 4: The regime switching strategy with the current month's equity premium as the threshold variable. This uses the same time frame as the GDP growth regime strategy; namely data starts on 1947:05 and forecasts start on 1957:01, with the end of the simulation at 2003:12. **Note:** $\bar{\varepsilon}_{os}$ is the average forecast error (the premium minus the forecast); a positive $\bar{\varepsilon}_{os}$ means that on average the premium was greater than the forecast. $\bar{\varepsilon}_{os,hm}$ is the average error for the ex-ante historical mean forecast.

Predictive Regressors	In Sample			Out of Sample Forecast			
	t -stat _{is} (growth)	t -stat _{is} (nongrowth)	adjusted R^2_{is}	$\Delta RMSE_{os}$	$\bar{\varepsilon}_{os}$	$\bar{\varepsilon}_{os,hm}$	R^2_{os}
Market Variables							
lagged dividend price ratio	1.67039	1.67478	2.08118%	0.00034%	-0.02292%	-0.31364%	0.01567%
lagged earnings price ratio	1.70776	1.13730	1.88060%	-0.01537%	-0.15326%	-0.31364%	-0.70909%
smoothed earnings price ratio	1.73907	1.20137	1.91799%	-0.00898%	-0.11247%	-0.31364%	-0.41400%
dividend payout ratio	0.09985	-1.26622	1.50237%	-0.00563%	-0.31082%	-0.31364%	-0.25963%
book to market ratio	1.26290	0.08390	1.50072%	-0.03741%	-0.24084%	-0.31364%	-1.73102%
net equity expansion ratio	0.08034	0.57828	1.31597%	-0.02954%	-0.33860%	-0.31364%	-1.36544%
Macroeconomic Indicators							
tbill rate	0.46348	3.20709	2.78493%	0.02238%	0.03936%	-0.31364%	1.02824%
long term yield	0.15379	2.30193	2.04178%	0.01086%	0.06264%	-0.31364%	0.49951%
long term return	0.03198	3.53540	3.06896%	-0.02606%	-0.48079%	-0.31364%	-1.20419%
term spread	0.67755	2.24479	2.06718%	-0.03076%	-0.58572%	-0.31364%	-1.42200%
default yield spread	-1.04179	-0.15548	1.42866%	-0.03740%	-0.44990%	-0.31364%	-1.73037%
default return spread	0.22141	0.24979	1.28228%	-0.03806%	-0.32799%	-0.31364%	-1.76110%
inflation (infeasible)	-0.09332	-2.14668	1.93963%	0.00193%	-0.29709%	-0.31364%	0.08893%
inflation adjusted	-1.40588	-1.83601	2.05379%	-0.00858%	-0.28980%	-0.32396%	-0.39569%

Table 5: The regime switching strategy with the current month's equity premium as the threshold, and imposing a nonnegativity constraint on the forecast. (Data starts 1947:05, forecasts begin 1957:01, and the simulation ends 2003:12). **Note:** $\bar{\varepsilon}_{os}$ is the average forecast error (the premium minus the forecast); a positive $\bar{\varepsilon}_{os}$ means that on average the premium was greater than the forecast. $\bar{\varepsilon}_{os,hm}$ is the average error for the ex-ante historical mean forecast.

Predictive Regressors	In Sample		Out of Sample Forecast			
	t -stat $_{t, is}$ (growth)	t -stat $_{t, is}$ (nongrowth)	adjusted $R^2_{t, is}$	$\Delta RMSE_{os}$	$\bar{\varepsilon}_{os}$	R^2_{os}
Market Variables						
lagged dividend price ratio	2.66793	-0.83012	2.11167%	-0.01226%	0.26578%	-0.43234%
lagged earnings price ratio	2.82872	0.75540	2.15857%	0.00528%	0.19514%	0.18612%
smoothed earnings price ratio	3.57635	0.11595	2.18810%	-0.00119%	0.36748%	-0.04193%
dividend payout ratio	-0.11393	2.16210	1.89352%	-0.01281%	-0.03493%	-0.45172%
book to market ratio	3.61585	-0.42499	2.25743%	-0.05454%	0.05829%	-2.59418%
net equity expansion ratio	0.78815	0.41474	0.72527%	-0.01823%	0.05223%	-0.86437%
Macroeconomic Indicators						
tbill rate	0.51174	1.55988	1.15424%	-0.01336%	0.30006%	-0.61834%
long term yield	0.44670	0.91647	1.03824%	-0.04888%	0.35082%	-2.22599%
long term return	0.23614	1.22021	0.82783%	-0.11281%	-0.00352%	-5.37620%
term spread	0.21458	1.73381	1.18936%	0.00113%	-0.01900%	0.05225%
default yield spread	-2.76371	2.07842	2.09038%	-0.04512%	0.06456%	-2.05381%
default return spread	-0.36320	1.68656	0.97985%	-0.04813%	0.03479%	-2.27665%
inflation (infeasible)	-0.11138	-0.75827	0.99342%	-0.03186%	0.05361%	-1.45018%
inflation adjusted	-1.17279	-0.05483	1.05559%	-0.02522%	0.08561%	-1.14700%

Table 6: The regime switching strategy with the current month's equity premium as the threshold variable. The time frame parameters are those of Campbell and Thompson. **Note:** $\bar{\varepsilon}_{os}$ is the average forecast error (the premium minus the forecast); a positive $\bar{\varepsilon}_{os}$ means that on average the premium was greater than the forecast.

Predictive Regressors	In Sample		Out of Sample Forecast			
	$t\text{-stat}_{t,s}$ (growth)	$t\text{-stat}_{t,s}$ (nongrowth)	adjusted $R_{t,s}^2$	$\Delta RMSE_{os}$	$\bar{\varepsilon}_{os}$	R_{os}^2
Market Variables						
lagged dividend price ratio	2.66793	-0.83012	2.11167%	0.00714%	0.11428%	0.25135%
lagged earnings price ratio	2.82872	0.75540	2.15857%	0.01950%	0.01173%	0.68579%
smoothed earnings price ratio	3.57635	0.11595	2.18810%	0.02506%	0.14786%	0.88087%
dividend payout ratio	-0.11393	2.16210	1.89352%	-0.00607%	-0.13443%	-0.21394%
book to market ratio	3.61585	-0.42499	2.25743%	-0.02226%	-0.06570%	-1.05481%
net equity expansion ratio	0.78815	0.41474	0.72527%	-0.00822%	0.02721%	-0.38922%
Macroeconomic Indicators						
tbill rate	0.51174	1.55988	1.15424%	-0.00231%	0.10750%	-0.10671%
long term yield	0.44670	0.91647	1.03824%	-0.01995%	0.08346%	-0.90580%
long term return	0.23614	1.22021	0.82783%	-0.04531%	-0.11968%	-2.14250%
term spread	0.21458	1.73381	1.18936%	0.00390%	-0.08026%	0.18017%
default yield spread	-2.76371	2.07842	2.09038%	-0.02895%	0.02091%	-1.31530%
default return spread	-0.36320	1.68656	0.97985%	-0.00823%	-0.06103%	-0.38764%
inflation (infeasible)	-0.11138	-0.75827	0.99342%	-0.01278%	0.00518%	-0.58056%
inflation adjusted	-1.17279	-0.05483	1.05559%	-0.00885%	0.04681%	-0.40163%

Table 7: The regime switching strategy with the current month's equity premium as the threshold variable. We impose a nonnegativity constraint on the value of the forecast. The time frame parameters are those of Campbell and Thompson. $\bar{\varepsilon}_{os}$ is the average forecast error (the premium minus the forecast); a positive $\bar{\varepsilon}_{os}$ means that on average the premium was greater than the forecast.