1. Recall that $\text{ACC}_r$ is the set of languages computable by polynomial-size constant depth circuits with $\land$, $\lor$, $\neg$ and $\text{mod}_r$ gates. Show that for any integers $r, s \geq 2$, if all of the prime factors of $r$ are factors of $s$ then $\text{ACC}_r \subseteq \text{ACC}_s$.

2. A $k$-sunflower is a collection of sets $S_1, \ldots, S_k$ such that for all $1 \leq i < j \leq k$, $S_i \cap S_j = S_1 \cap S_2$. Show that for any collection of $(k - 1)^r! + 1$ sets of size at most $r$ there is a subcollection of $k$ sets that form a sunflower.

3. A function $f : \{0, 1\}^n \to \{0, 1\}$ is a slice function if there is an $k$ such that there are no $x$ with less than $k$ ones and $f(x) = 1$ and no $x$ with more than $k$ ones with $f(x) = 0$.

Show that if a slice function $f$ has polynomial-size circuits then $f$ has polynomial-size monotone circuits.

Hint: You will need that there are polynomial-size monotone circuits for majority. Create a sorting network and convert that to a monotone circuit.

4. Show that for any constant $c$, there is a language $L$ in $\Sigma_1^n$ such that $L$ does not have circuits of size $n^c$. Can you put $L$ in $\Sigma_2^p$?