1. Let $A$ be in P and $p$ be a polynomial. Define

$$\text{Leaf}_A(x) = a_1 a_2 \ldots a_{2^p(|x|)}$$

where $a_i = 1$ if $(x, i) \in A$ and 0 otherwise.

Show that $L$ is in PSPACE if and only if there is a regular language $R$, a set $A$ in P and a polynomial $p$ such that

$$x \in L \Leftrightarrow \text{Leaf}_A(x) \in R.$$  

Hint: Use Barrington’s Theorem.

2. Show that $L$ is in $C_P \cap \text{coC}_P$ iff there exist GapP functions $f$ and $g$ such that for all $x$, $g(x) \neq 0$ and

- If $x$ is in $L$ then $f(x) = g(x)$, and
- If $x$ is not in $L$ then $f(x) = 0$.

3. Show that $L$ is in PP iff there exist GapP functions $f$ and $g$ such that for all $x$, $g(x) \neq 0$ and

- If $x$ is in $L$ then $2g(x)/3 \leq f(x) \leq g(x)$, and
- If $x$ is not in $L$ then $0 \leq f(x) \leq g(x)/3$.

4. Show that the decision tree complexity of a Boolean function $f$ is bounded by the product of its certificate complexity and block sensitivity. Show that this implies that the decision tree complexity of $f$ is bounded by $O(d^6)$ where $d$ is the degree of the approximating polynomial of $f$. This is the best known upper bound.