1. (4 points) Recall: a Sperner family on $n$ elements is a set of pairwise incomparable subsets of $[n] = \{1, \ldots, n\}$. (“Incomparable” means neither is a subset of the other.) True or false: every maximal Sperner family on $n$ elements is maximum. Prove your answer.

2. (4 points) Recall: a hypergraph on the vertex set $V$ is a set of subsets of $V$. Count the hypergraphs on a given set $V$ of $n$ vertices. Give a simple closed-form expression. Do not prove.

3. (4 points) Recall: the generating function of the sequence $a_0, a_1, \ldots$ is the function $f(x) = \sum_{n=0}^{\infty} a_n x^n$. Give closed-form expressions of the generating functions of the sequences
   
   \begin{align*}
   (a) & \quad a_n = 1/(n + 1) \\
   (b) & \quad b_n = \binom{100}{n} 2^{-n}
   \end{align*}

4. (3 points) Recall: $\pi(x)$ denotes the number of primes $\leq x$. State the Prime Number Theorem. Define the symbol involved in the statement.
5. (15 points) Prove the BLYM inequality (called LYM(B) in last class): If \( \{A_1, \ldots, A_m\} \) is a Sperner family of subsets of \([n]\) then

\[
\sum_{i=1}^{m} \frac{1}{\binom{n}{|A_i|}} \leq 1.
\]