Exercise 1. Prove: the dimension of the space of homogeneous polynomials of degree \( s \) in \( n \) variables is \( \binom{n+s-1}{s} \). (A multivariate polynomial is homogeneous if it is a linear combination of monomials of the same degree. Example: \( x^4 + 5x^3y + 13x^2yz + xz^3 \) is a homogeneous polynomial of degree 4 in 3 variables. The polynomial \( x^4 + xyz \) is not homogeneous.)

Hint: Count the monomials of degree exactly \( s \) in \( n \) variables.

Exercise 2. Prove: if \( \mu_1, \ldots, \mu_m \in \mathbb{F}^n \), \( f_1, \ldots, f_m \in \mathbb{F}[x_1, \ldots, x_n] \), and \( f_i(\mu(j)) \neq 0 \) if \( i = j \) and \( f_i(\mu(j)) = 0 \) if \( i > j \), then \( f_1, \ldots, f_m \) are linearly independent. (Note: we have no condition for the case \( i < j \).)

Exercise 6. Prove the Frankl-Wilson Theorem by adapting the proof given in class to the case of non-uniform set systems. Hint: Use Exercise 2. The polynomials used in class will have to be modified slightly, and put in the correct order.

Exercise 7. Prove the Ray-Chaudhuri–Wilson Theorem. Hint: Use Exercise 2. The polynomials used in class will have to be modified slightly, and put in the correct order.
Let $\overline{h_I}$ be the multilinearization of $h_I$, and let $\overline{\mathcal{F}_1}, \ldots, \overline{\mathcal{F}_m}$ be the multilinear polynomials from the proof given in class. Use Exercise 2 to prove that the following set of polynomials is linearly independent: $\overline{\mathcal{F}_1}, \ldots, \overline{\mathcal{F}_m}$, together with $\overline{h_I}$, for all $I \subset n$ such that $|I| \leq s - 1$. Deduce that $m + \binom{n}{\leq s - 1} \leq \binom{n}{\leq s}$, and so $m \leq \binom{n}{s}$.