1. Sketch the region $R$ defined by $0 \leq y \leq 1/x^3$, $1 \leq x \leq 2$.
   a) Find (exactly) the number $a$ such that the line $x = a$ divides $R$ into two parts of equal area.
   b) Then find (to 3 places) the number $b$ such that the line $y = b$ divides $R$ into two parts of equal area.

2. Which has more area, the region in the first quadrant enclosed by the line $x + y = 1$ and the circle $x^2 + y^2 = 1$, or the region in the first quadrant enclosed by the line $x + y = 1$ and the curve $\sqrt{x} + \sqrt{y} = 1$? Justify your answer.

3. Let $R$ be the parabolic region in the $x$-$y$ plane bounded below by the curve $y = x^2$ and above by the line $y = 1$.
   a) Sketch $R$. Set up and evaluate an integral that gives the area of $R$.
   b) Suppose a solid has base $R$ and the cross-sections of the solid perpendicular to the $y$-axis are squares. Sketch the solid and find its volume.
   c) Suppose a solid has base $R$ and the cross-sections of the solid perpendicular to the $y$-axis are equilateral triangles. Sketch the solid and find its volume.

4. Start with the region $A$ in the first quadrant enclosed by the $x$-axis and the parabola $y = 2x(2-x)$. Then obtain solids of revolution $S_1$, $S_2$, and $S_3$ by revolving $A$ about the lines $y = 4$, $y = -2$, and $x = 4$ respectively. All three solids are (unusual) “doughnuts” which are 8 units across, whose hole is 4 units across, and whose height is 2 units. Sketch them.
   (a) Which do you expect to have larger volume, $S_1$ or $S_2$? Compute their volumes exactly and check your guess.
   (b) Compute the volume of $S_3$. (It may be harder to guess in advance how $S_3$ compares in volume to $S_2$ and $S_1$.)