1. A liquid whose density is 300 kg/m³ fills three buried containers. Each container, illustrated below, is 10 meters tall. The top of each container is at ground level. All three containers have the same volume. The middle container is a cylinder, and the other two are circular cones. Which container needs the least amount of work to empty (that is, to pump the liquid to ground level)? Which container needs the most work to empty? Justify your assertions by computing the work necessary in each case*.

![Containers Diagram](image)

Note: the volume of a circular cylinder with base radius $r$ and height $h$ is $\pi r^2 h$; the volume of a circular cone with base radius $r$ and height $h$ is $\pi r^2 h/3$.

2. Electrons repel each other with a force which is inversely proportional to the square of the distance between them; call the proportionality constant $k$ in the units to be used. Suppose one electron is fixed at $x = 0$ on the $x$-axis.
   a) Find the work done in moving a second electron along the $x$-axis from the point $x = 10$ to the point $x = 1$.
   b) Find the work done in moving the second electron along the $x$-axis from the point $x = M$ to the point $x = 1$.
   c) What happens to your answer in b) (which should depend on $M$) as $M \to +\infty$?

3. Suppose that the outdoor temperature (in °F) on a particular day was approximated by the function
   \[ T(t) = 50 + 14 \sin \frac{\pi t}{12}, \]
   where $t$ is time (in hours) after 9 am.
   a) Find the maximum temperature $T_{\text{max}}$, minimum temperature $T_{\text{min}}$, and average temperature
   \[ T_{\text{aver}} = \frac{1}{12} \int_0^{12} T(t) \, dt \]
   on that day during the period between 9 am and 9 pm.
   b) Show that $T_{\text{aver}} \neq \frac{1}{2}(T_{\text{min}} + T_{\text{max}})$ (This is the definition that the weather bureau uses for “average temperature”).

* You probably should begin with the cylinder.
c) Show that if $T$ is not given by the above formula, but rather $T(t)$ is a linear function of $t$, then $T_{\text{aver}} = \frac{1}{2}(T_{\text{min}} + T_{\text{max}})$. (Use either geometric reasoning or an integral.)

4. Calculate four of the following integrals:

\[
\int x \cos x^2 \, dx \quad \int x^2 \cos x \, dx \quad \int x^2 \cos x \, dx \quad \int x^2 \cos^2 x \, dx \quad \int x \cos^2 x \, dx
\]