1. If \( n \) is sufficiently large, the following functions of \( n \) fall in a definite increasing order, so that each function is very much larger than the one that precedes it. List the functions below in order of size from smallest to largest.

\[ n, \ n^n, \ \ln n, \ 4^n, \ 2^n, \ n \ln n, \ 2^{n^2}, \ \sqrt{n^6 + 1}, \ (n^3 + 1)^{2/3} \]

Where would \( n! \) fit in this list? Explain.

2. Under the hypotheses of the integral test, if \( a_n = f(n) \) and \( s_n = a_1 + a_2 + \cdots + a_n \), then

\[ \int_1^n f(x) \, dx \leq s_n \leq a_1 + \int_1^n f(x) \, dx \quad \text{for each positive integer } n. \]

In the case of the harmonic series \( \sum_{n=1}^\infty \frac{1}{n} \), this means that

\[ \ln n \leq 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \leq 1 + \ln n \quad \text{for each positive integer } n. \]

a) Find the analogous inequalities for the series \( \sum_{n=1}^\infty \frac{1}{\sqrt{n}} \) and for the series \( \sum_{n=2}^\infty \frac{1}{n \ln n} \).

b) Estimate the sum of the first \( 10^{10} \) terms of the series, in each case. Then estimate the sum of the first \( 10^{100} \) terms.

c) Of the three series, which diverges the fastest? the slowest?

3. Use the comparison or limit comparison test to decide if the following series converge.

\[ \sum_{n=1}^\infty \frac{4 - \sin n}{n^2 + 1} \quad \sum_{n=1}^\infty \frac{4 - \sin n}{2^n + 1} \quad \sum_{n=1}^\infty \frac{\sqrt{n + 3}}{n^3 + 2} \]

For the series which converge(s), give an approximation of its sum, together with an error estimate, as follows. First calculate the sum \( s_5 \) of the first 5 terms. Then estimate the “tail” \( \sum_{n=6}^\infty a_n \) by comparing it with an appropriate improper integral or geometric series.