[Pap23a] Theodoros Papamakarios. Depth-d frege systems are not automatable unless P=NP. *Electronic Colloqium on Computational Complexity, Report No. 121*, 2023.

This paper shows that bounded depth Frege systems are NP-hard to automate, extending previous results showing the same for resolution, cutting planes, Res(k), and various algebraic proof systems. Like the previous proofs, the reduction from SAT starts with a form of the reflection principle for resolution, which is then lifted combining relativization and substition of variables with Sipser functions. The main technical tool for proving the lower bound is a combination of a form of the Frust-Saxe-Sipser switching lemma, and a variation of the small restriction switching lemma of Segerlind, Buss and Impagliazzo used previously by Garlík.

[Pap23b] Theodoros Papamakarios. A super-polynomial separation between resolution and cut-free sequent calculus. In *Proceedings of the 48th International Symposium on Mathematical Foundations of Computer Science*, volume 272, pages 74:1–74:15, 2023.

This paper shows a super-polynomial separation between resolution and cut-free sequent calculus for refuting sets of clauses, answering a question which appears already in the 1974 paper of Cook and Rechow. It is well known that adding cuts (even of low complexity, e.g. bounded depth formulas or formulas of at most $k > 1$ variables) in cut-free sequent calculus, gives exponentially shorter proofs; this paper shows that even adding propositional variables as cuts gives super-polynomially shorter proofs. It does this by considering, for the first time, the width of sequent calculus proofs, and giving a combinatorial characterization of sequent calculus width, generalizing the analogous characterization known for resolution width. Using that characterization, a quadratic gap is shown between resolution and cut-free sequent calculus width. The latter turns out, it is argued, to be a quite useful notion; in particular, the quadratic width gap, apart from the super-polynomial size gap, also gives a quadratic gap between resolution width and monomial space—no gap between the two was known before.

[PR23] Theodoros Papamakarios and Alexander Razborov. Space characterizations of complexity measures and size-space trade-offs in propositional proof systems. *Journal of Computer and System Sciences*, 137:20–36, 2023.

This paper identifies two new clusters of proof complexity measures, each characterized by a space measure. The first cluster is characterized by reguralized clause and monomial space, which enables us first to re-derive and strengthen previously known space lower bounds, and secondly to rephrase the existence of strong trade-offs between clause space and size in resolution in terms of separating clause space from the logarithm of tree-like size. The second cluster is characterized by Σ_2 space and reguralized Σ_2 space, and this implies that once we go up to proof systems of depth 2, and in fact already a bit below depth 2, strong size-space trade-offs are, surprisingly, ruled out. Whether strong size-space

trade-offs exist for systems of intermediate depth is an important question that remains open. A new complexity measure for resolution, called positive depth, is introduced, and it is shown that tree-like resolution proofs can be seen as describing strategies in a pebble game, with proof size corresponding to number of steps and positive depth to number of pebbles in the pebble game. Using this characterization, a quadratic lower bound on tree-like resolution size is shown for formulas refutable in clause space 4.